WHY SHALL WE ALSO TEACH THE THEORY BEHIND ENGINEERING GRAPHICS

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WHY SHALL WE ALSO TEACH THE THEORY BEHIND ENGINEERING GRAPHICS[1)]

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ABSTRACT: With regard to our graphics courses this is a pleading for emphasizing the theoretical aspects more intensively, regardless of whether these are of actual applicability or not. At the same time this is a late contribution to the special forum on "Modernization of Graphics Education", which took place at the 6th ICECGDG in Tokyo 1994.

1. INTRODUCTION

At our last conference in Tokyo in 1994 there had been a special forum on "Modernization of Graphics Education". I had been one of the panelists, and in my introductory statement (10) I defined what in Austria – or better in central Europe – is understood under "Descriptive Geometry" and why also a certain amount of theory should be included. I remember that the following discussion revealed some concern in the audience which could be summarized in the question: What should too much theory be good for?

It is one of the great advantages that our truly international conferences bring together people from various traditions and with very different opinions. This is always a good stimulation to reflect upon the own position and either to defend or to reject it. In my case I am going to defend my point of view, now, two years after, and here, from this beautiful stage of the Słowacki Theatre.

2. THE SPUTNIK EFFECT

One remark at the beginning in order to avoid some misunderstanding. In central Europe the subject "Engineering Graphics" is traditionally devided into two parts (cf. EDER(4)). This holds for the curricula for mechanical engineering as well as for constructional engineering and architecture:

One part deals with geometry and the theory of graphics and this is usually called *"Descriptive Geometry"*. It is taught either by engineers or by mathematicians. Here the projection theory is combined with more intuitively presented concepts of spatial geometry like 3d-relationships, modeling, curves and surfaces, intersections and developments, 3d-geometry problem solving.

The other part is *"Engineering Design"* which traditionally is taught by engineers with industrial experience. Here the aspects of designing are treated like standards, dimensioning, documentation and conventions for assembly and manufacture.

In the following I'm adressing the "Descriptive Geometry" only.

In the last decades graphics educators faced worldwide the problem that their subject came into discussion and their courses were reduced. Since then it was a successful defense strategy to demonstrate that we teach something that is of practical importance. Therefore we all started to cancel parts of the theory whenever the applicability wasn't apparent. Exercises where selected from real applications only. We began to reject many examples with impressive ideas and beautiful pictures but that seemed to be of no importance for the world of engineering. This is very similar to a phenomenon in the U.S.A. that usually has been called *"Sputnik effect"*.

3. PLEADING FOR MORE THEORY

Our strategy of repressing the theory against applicability was right as we all tried to keep our subject alive. But what means applicability? We believed that we were able to estimate which parts of geometry are needed – at present. But for the future? Note, that we have to educate engineers for presence and future[2)].

And what is education? Roughly spoken, education

[1)] Invited lecture, presented at the Opening Session of the 7th ICECGDG in Cracow 1996. This paper should fill the blanks in the Proceedings, Vol. 1, p. 7-8.

[2)] I recommend everybody in the audience to study the "conceptual model on the engineering graphics curriculum for the year 2000" proposed by G. BERTOLINE et al.(2).

means all that what remains in the student's mind after he has forgotten most parts of information that had been filled into their brains.

What could remain from our graphics courses? There should remain the trained skills like practice in visualization, manual skills for sketching and for manipulating the medium. But I expect that our courses additionally bring about

- an impression of the power of visualization,
- an imagination of the ideal world of geometry, its teminology and concepts in breadth and depth,
- a feeling for logical rigidity, and perhaps
- sensitivity to the beauty of geometric reasoning,
- creativity and an open mind to new ideas.

I personally didn't behave different to the majority of graphics educators in the past. I also restricted my courses to subjects that had apparently been applicable. In the meantime I'm beginning to change my mind. Perhaps, I am going to understand S. SLABY better (cf. his position papers (8) and (9)). Or it's just a consequence of becoming elder.

Why not select topics for our courses that meet our general educational aims better, which are easier to grasp and more impressive, which better lend wings to the students' creativity (cf. MEIRER(6))? Why should not methodology and didactics govern our lectures and exercises?

We tend too much to ask for applicability. What is the applicability of any outstanding masterpiece of art like Leonardo DA'VINCI's "Mona Lisa" or W.A. MOZART's "Magic Flute"? What is the practical use of any spectacular athletic performance like a new world record in broad jump? *Geometry is an intellectual art of mankind and a particular kind of creativity of mind.*

Nobody knows what will be of importance for engineers in the future, or even in the near future! In the following I want to present an example which should be convincing in this sense:

4. THE APOLLONIAN CIRCLE PROBLEM AND THE GLOBAL POSITIONING SYSTEM

4.1 Apollonian Problem

The truly classical circle problem of APOLLONIUS OF PERGE (about 260-190 B.C.) reads: Find all circles which touch three given circles c_1, c_2, c_3 (Fig. 1). It is remarkable that this problem can be solved exactly with compass and straight edge – even for each dimension (cf. HOHENBERG(5)) – but the construction of the atmost 8 different solutions needs some sophisticated ideas, for example the stereographic projection or the cyclographic projection (see e.g. (1), (3), (7) or ZSOMBOR-MURRAY(12)). Just the 3d-version of this problem had recently its unexpected rebirth in the field of satellite-geodesy. Until recent nobody of us would have recommended to spend time for this old and completely solved problem of APOLLONIUS in geometry courses.

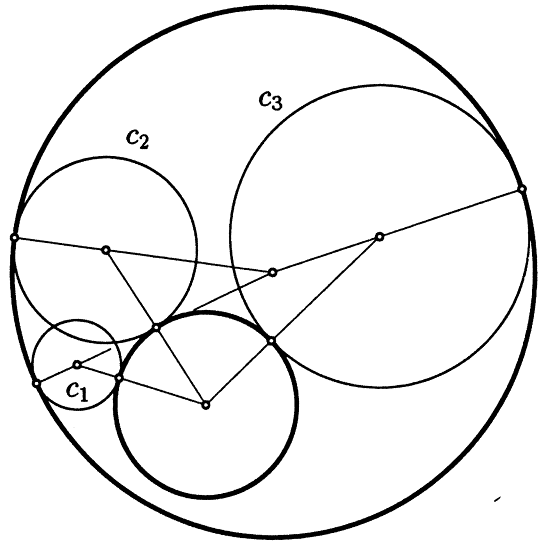

Figure 1: Apollonian circle problem, c_1, c_2, c_3 given

4.2 Global Positioning System (GPS)

What does *Global Positioning System (GPS)* mean? At a cost of about U.S.$ 2.000 everybody can buy an instrument which looks like a pocket computer. Push a button, and a few seconds later you get the coordinates of your position, up to less than half a meter. And this works, wherever you are on earth, except in some neighborhood of the poles.

This system (see e.g. Th. WUNDERLICH(11)) had been developed by the U.S. army[3]). It is based on 17 satellites that permanently sorround the earth in such a way that for almost each position on earth there are at least 4 satellites available over the horizon (Fig. 2). Each satellite sends permanently a message that contains the satellite's exact instantaneous location and the exact time when the message is sent off. There are stations on earth which synchronize the satellites' clocks and which keep each satellite informed about its path parameters.

Your pocket instrument at position $\mathbf{p}$ on earth receives at the same time the messages from at least four satellites with coordinate vectors $\mathbf{s}_1, \ldots, \mathbf{s}_4$ (Fig. 2). From the time difference between sending and receipt of each single message the distance $||\mathbf{s}_i - \mathbf{p}||$ is computed, but only up to a constant c. This constant is caused by the fact that the clock

[3]) It is now accessable for the public. But intentional noise has been reducing the precision ("Selective Availability") – except at the time of the Golf War.

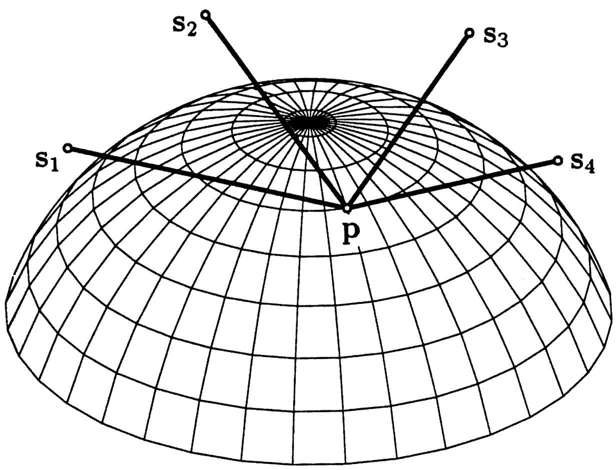

Figure 2: Global Positioning System (GPS)

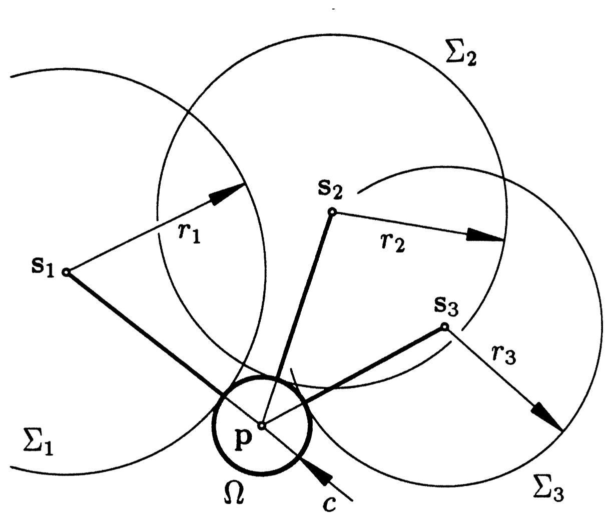

Figure 3: GPS and the Apollonian sphere problem

in your pocket receiver is not absolutely synchronous with that in the satellites. If your clock is a little bit slow, then the measured time difference looks shorter, and the same holds for the computed distance. This means we measure only the socalled *pseudoranges*

$$r_i := \|\mathbf{s}_i - \mathbf{p}\| - c \quad \text{for} \quad i = 1, \ldots, 4,$$

and c and the three coordinates of your position $\mathbf{p}$ are the four unknowns in our problem.

Now imagine that for $i = 1, .., 4$ the pseudorange r_i is the radius of a sphere Σ_i centered at $\mathbf{s}_i$ (see Fig. 3). Then the sphere Ω with center $\mathbf{p}$ and radius c must touch the four spheres $\Sigma_1, \ldots, \Sigma_4$. Hence, one of the atmost 16 different solutions[4]) of the Apollonian sphere problem for given $\Sigma_1, \ldots, \Sigma_4$ is centered exactly at the required position $\mathbf{p}$. As Ω touches all Σ_i either from outside (c positive) or from inside (c negative), there are only two solutions remaining. In general the solution with the smaller radius c will be the right one. If more than four satellites are available at the same time, then the accuracy can be improved by applying adjustment methods.

But this is still not the whole story: Your pocket receiver presents also a factor *GDOP (Geometric Dilution of Presison)* that indicates the precision of computation. The higher the GDOP, the worse the result. What is the worst case?

4.3 Critical Configurations

Solving the Apollonian sphere problem numerically means reducing the system of 4 nonlinear equations to one equation only. But the computation of the solutions of this equation fails considerably when there is a zero of higher multiplicity, to say when two solutions happen to coincide. How to characterize such *critical configurations*?

The set of points $\mathbf{p}$ with constant difference of distances

$$\|\mathbf{s}_i - \mathbf{p}\| - \|\mathbf{s}_1 - \mathbf{p}\| = r_i - r_1$$

equals one sheet of a two-sheet hyperboloid of revolution Φ_{i1} with focal points at $\mathbf{s}_i$ and $\mathbf{s}_1$ (Fig. 4). Hence the required position $\mathbf{p}$ is common for the three hyperboloids Φ_{21}, Φ_{31} and Φ_{41}. In the critical case the three tangent planes at $\mathbf{p}$ have a line t in common. This is the *line of uncertainty* that passes through two coinciding solutions.

The tangent plane τ of Φ_{i1} at $\mathbf{p}$ bisects the angle between the lines connecting $\mathbf{p}$ and the focal points. Therefore the reflection in τ exchanges the two connections $\mathbf{s}_i\mathbf{p}$ and $\mathbf{s}_1\mathbf{p}$ while the tangent line t is preserved. This implies equal angles

$$\sphericalangle\, \mathbf{p}\mathbf{s}_1 t = \sphericalangle\, \mathbf{p}\mathbf{s}_2 t = \sphericalangle\, \mathbf{p}\mathbf{s}_3 t = \sphericalangle\, \mathbf{p}\mathbf{s}_4 t,$$

and we conclude: *The configuration is critical if and only if the four satellites $\mathbf{s}_1, \ldots, \mathbf{s}_4$ are located on a cone of revolution with the apex at $\mathbf{p}$ and with axis t.*

For obtaining the equivalent analytic expression we intersect the line segments joining $\mathbf{p}$ and $\mathbf{s}_i$ with the unit sphere Ψ centered at $\mathbf{p}$ (Fig. 5). Then just in the critical case the endpoints are located on the intersection curve between Ψ and the (half)cone of revolution, i.e. on a circle. Hence for the four points

$$\mathbf{e}_i := \frac{\mathbf{s}_i - \mathbf{p}}{\|\mathbf{s}_i - \mathbf{p}\|}, \quad i = 1, \ldots, 4$$

the volume of the spanned tetrahedron vanishes, i.e.

$$\det(\mathbf{e}_2 - \mathbf{e}_1, \mathbf{e}_3 - \mathbf{e}_1, \mathbf{e}_4 - \mathbf{e}_1) = 0.$$

The mentioned configuration factor GDOP turns out to be proportional to the reciprocal of this volume (for more details see (11)).

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[4]) It can even happen that there is a continuum of different solutions. In this exceptional case the four spheres are generators of the same kind for any DUPIN's cyclide.

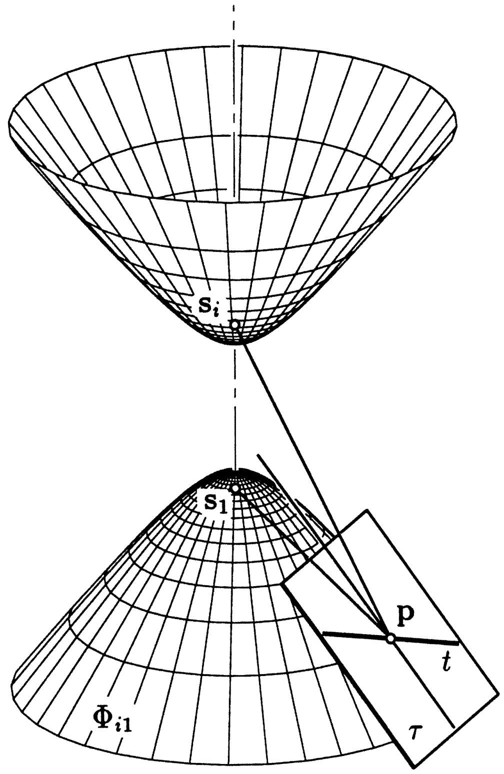

Figure 4: Hyperboloid Φ_{i1}, $\sphericalangle\, \mathbf{ps}_i\, t = \sphericalangle\, \mathbf{ps}_1\, t$

What we see at this example, that also takes place in many other fields of natural or technical sciences: Behind the theory there is frequently a simple geometric model, a certain kind of geometric idealization. Remember EINSTEIN's theory of special relativity. Therefore enhancing the students' capability for geometric reasoning brings about a better understanding for all these various geometric models, too.

5. CONCLUSION

We all certainly agree that the best way of comprehensing engineering graphics is *"learning by doing"*. And this doesn't depend on the applicability of the objects we are dealing with. And it doesn't depend on the tool, too. *We have to educate and not to produce!* Therefore don't be afraid of theory! Seize the chance to demonstrate the power and the beauty of geometry.

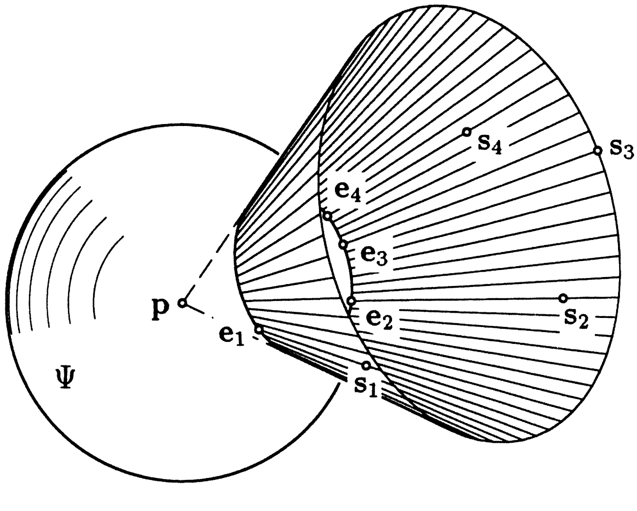

Figure 5: Critical configuration

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