Recent Applications of Descriptive Geometry to Mechanical Engineering

Hellmuth STACHEL
Vienna University of Technology, Vienna, Austria

ABSTRACT: This is a brief summary of a paper presented at the 5th Japan-China Joint Conference on Graphics Education in Osaka 2001.

1. INTRODUCTION

At Austrian universities the courses on Descriptive Geometry are not only confined to pure graphical methods, but they offer a visually based introduction to spatial geometry. Furthermore, it is an Austrian tradition that the lectures on the "Theory of Mechanisms" for students of mechanical engineering are held by the same persons who teach Descriptive Geometry. As a consequence, in Descriptive Geometry emphasis is also laid on kinematical aspects. However, on has to be aware that in Kinematics not only graphical methods but also some mathematics are necessary, in particular basics of Linear Algebra and Differential Geometry.

One advantage of this combination of Descriptive Geometry and Kinematics in Austria is that my institution, the Institute of Geometry at the Vienna University of Technology is frequently faced with real world problems presented by different Austrian companies. Recently we were involved into the design of a swab folding machine and a needle punching machine. In both cases the intention to speed up the production process caused kinematical problems. This is always a challenge for the stuff and inspiring for their scientific work. And these problems offer the chance to make the lectures and exercises in Descriptive Geometry as well as in Kinematics more vivid and perhaps more motivating for students. Needless to say that in any of these consulting activities computer support is necessary, either with professional CAD-software of – in most of the cases – software developed at our insitution.

In the following a very few examples will be presented where Descriptive Geometry in the more general sense as explained above is most useful or even inevitable for the understanding of mechanisms or for the solution of engineering design problems.

2. PARALLEL MANIPULATORS

Product development – e.g. in the high volume production processes within the automotive industry – affects the current production technologies dramatically. Especially function integration in so-called "integral components" leads to more complex geometries and to the need of improved task oriented solutions for the entire production processes. In the last decades the concept of frame-construction with a serial arrangement of axes *(serial roboters)* has been developed.[1] However, a substantial disadvantage of these structures are the bending forces and the large amount of moved masses. It is a characteristic property of serial machines that drives have to carry the mass of other drives and axes which reduces the system's ability of dynamic movement.

A promising chance for improving the dynamics of machine tools is particularly seen in the design of parallel-kinematics based on a bar linkage, the so called Hexapod-concept. Various different parallel-kinematics are on the market, those with lenght-variable legs – e.g., Stuart-Gough-platform (Fig. 1), or others with legs of fixed length. An example for the latter type is a machine tool for drilling and milling operations with three-axes parallel kinematics (Fig. 2) available at the Institute of Production Engineering, TU Vienna. Here the translatory movement of the end-effector is controlled by three joints which run on three parallel lines.

The simplest geometric model of this special tool (see the detail in Fig. 3) consists of three edges of given lenghts which meet at a common point, the

[1] From the educational point of view it is very useful to let students analyse the geometric structure of serial robots, the relative motions between adjacent systems, and the workspace. An interesting problem is also to determine on how many different ways one given position of the end-effector can be reached. One of these examples has been presented in FUHS and STACHEL (2).

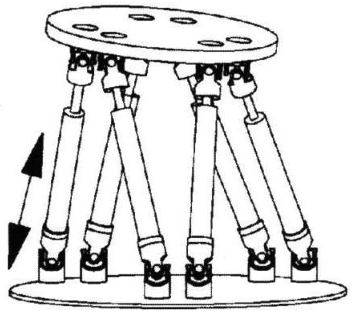

Figure 1: Example of a parallel manipulator

tool center point TCP, which can be seen als the apex of a three-sided pyramid. The base triangle of this pyramide is variable; the three vertices can be specified on three parallel lines, respectively.

This model allows immediately to analyse this manipulator from the geometric point of view: The problem of em forward kinematics, i.e., find the position of the TCP for given positions of the three base joints, is equivalent to the intersection of three spheres. It is evident that no "singular positions" with two coinciding solutions can appear. This geometric model makes it also rather easy to define the workspace or to solve the problem of *inverse kinematics*, i.e., for a given location of the TCP find the base joint positions.

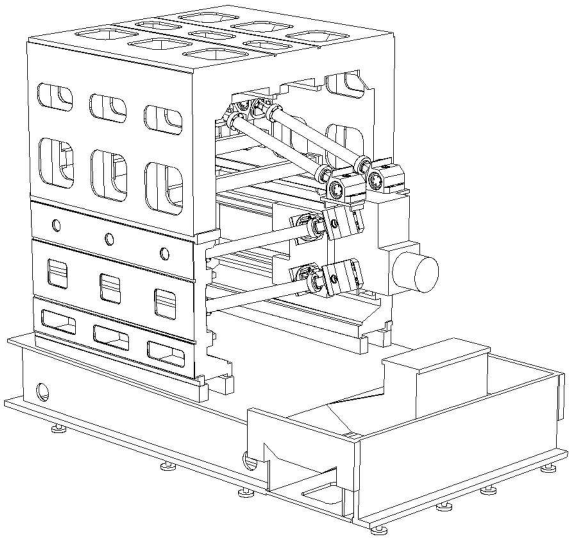

Figure 2: A three-axes version of a parallel manipulator for translatory movement

However, production and mounting tolerances effect that the real machine corresponds only partly to the ideal geometric model. Hence there are many problems left when this machine tool needs to improve its positioning precision, e.g., down to 0.01 mm, when the length of the legs is about 1 m.

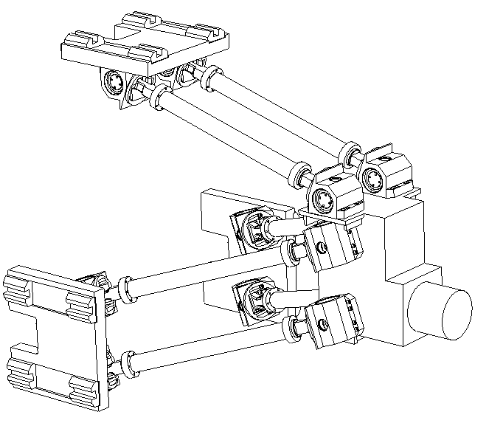

Figure 3: The skeleton of this manipulator

For this purpose, on the one hand precise measurements for the precisely determining the misplacement of the end effector in different positions. On the other hand, one has to develop strategies to compensate these erros. In particular one must figure out which deviations of the ideal model are most responsible for the misplacements in different areas. This needs sophisticated methods from Linear Algebra and in particular the Jacobi transformation which maps the derivations of the input parameters (coordinates of the base joints) onto the instantaneous movement of the end-effector. This is part of a contemporary joint project where I am involved.

3. THE UNIVERSAL JOINT

The second example has to do with the transmission of rotations about nonparallel axes a_{10}, a_{20}.

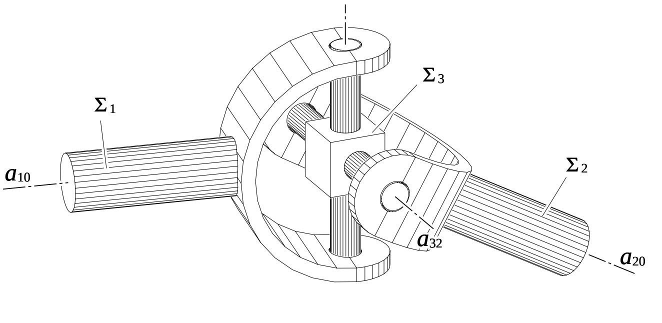

Figure 4: The cardan joint

In STACHEL (1) I presented a *cardan joint* (Fig. 4) as an example where basic Descriptive Geometry methods (auxiliary views and right-angle-theorem) allow to grasp easily why this transmission is not uniform, to say, that the ration between output and input angular velocity is not constant. This is the reason why in motor cars for driving the wheels another type of transmission is

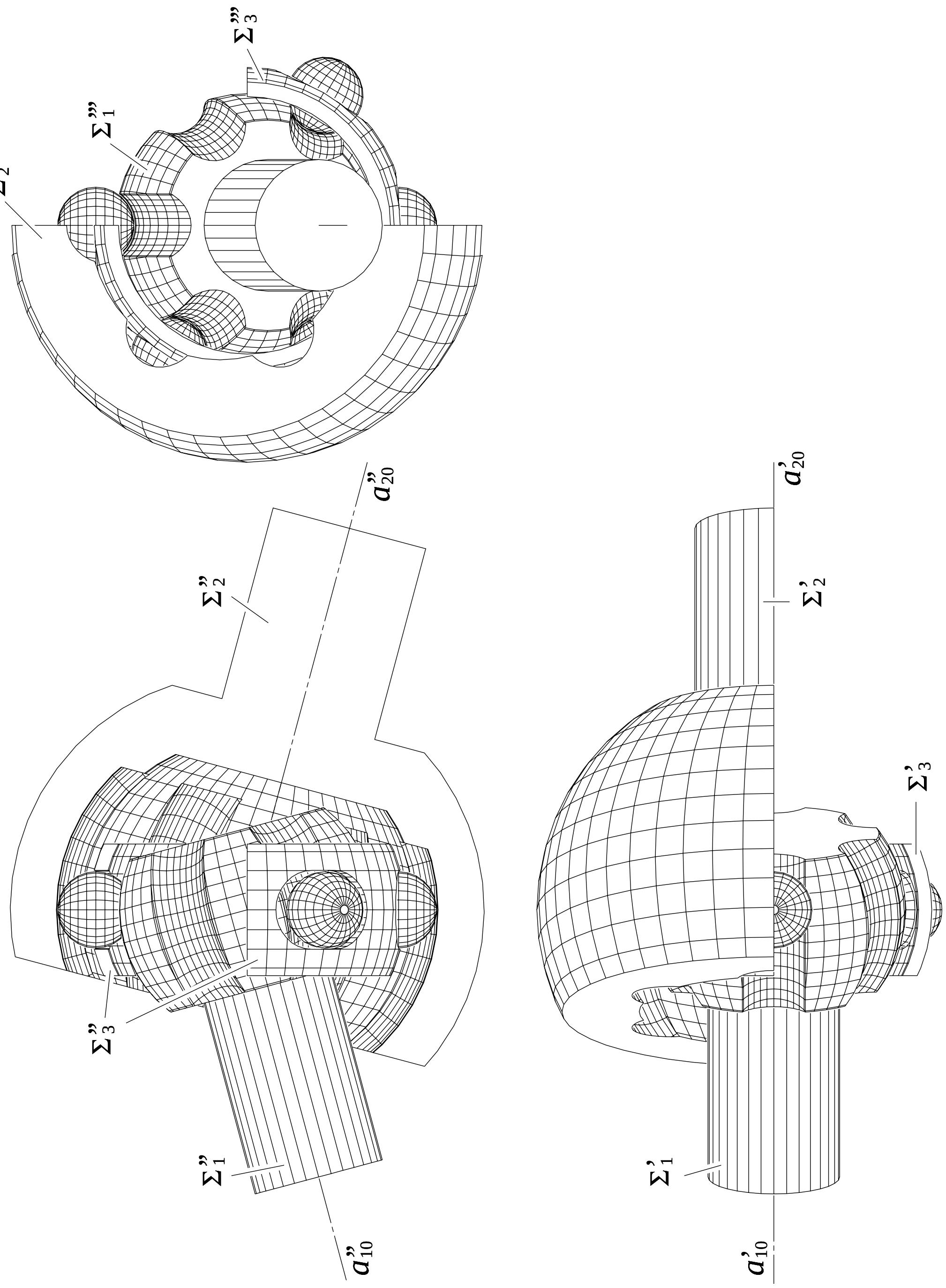

Figure 5: The geometric structure of the unversal joint

used, the *universal joint*. Actually, this is not a recent development but dates back to inventions bei C.W. WEISS (1925, US.pat.no. 1,522.351) and A.H. RZEPPA (1935, US.pat.no. 2,010.899).

Though there is much interesting geometry included, this kind of transmission seems to be rather unknown in the scientific community of graphics educators.

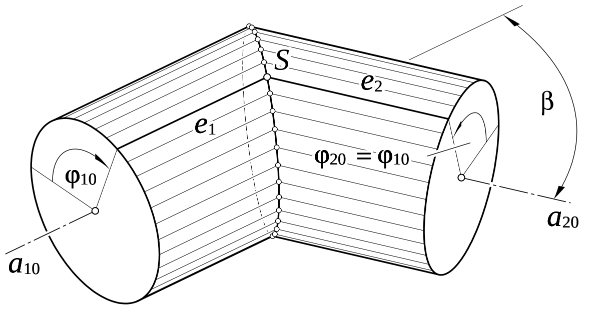

Figure 6: The condition for uniform transmission

How to obtain a *uniform transmission* from the input rotation about the axis a_{10} to the output rotation about a_{20}? Suppose, these are the axes of cylinders of revolution with the same radius r (Fig. 6). On these cylinders, take the generators e_1, e_2, respectively, both located in the plane spanned by the two axes. Then these generators intersect at a point of a bisector plane σ of a_{10} and a_{20}. And under a uniform transmission, i.e., both cylinders rotate through the same angle $\varphi_{20} = \varphi_{10}$, the generators are permanently intersecting and the point S of intersection remains in the plane σ of symmetry.

S traces an ellipse. Therefore S is neither attachable to in the input system Σ_1 nor to the output system Σ_2. Furthermore, all these points of intersection do not belong to a common rigid system as their mutual differences change (see auxiliary view in Fig. 7).

Another formulation of the condition for uniform transmission reads:
At any instant the radial planes $\mu_1 = a_{10}e_1$ *and* $\mu_2 = a_{20}e_2$ *must intersect in a line of* σ.

At the unversal joint (Fig. 5)[2] this condition is met by (six) steel balls with centers remaining in the plane σ and preserving the distance ρ to the point $O = a_{10} \cap a_{20}$ of intersection. Hence the centers of these spheres are located on a circle with radius ρ. The spheres can be combined by a sort of ball bearing Σ_3. In Fig. 5 this ball bearing is displayed with a quarter section.

The centers of the steel balls remain with respect to Σ_1 in planes $\mu_1 \supset a_{10}$. Therefore they are movable on circles. This is materialized by longitudinal grooves at the end of the input part. The output part Σ_2 is designed as a hollow sphere embracing the end of Σ_1 and all balls in Σ_3. Again, toroidal grooves – this time on the inside of Σ_2 – guarantee that the centers of the steel balls are held in radial planes μ_2 with respect to Σ_2.

[2] This figure has been produced with the 3D-modelling software CAD-3D developed at the Institute of Geometry, Vienna.

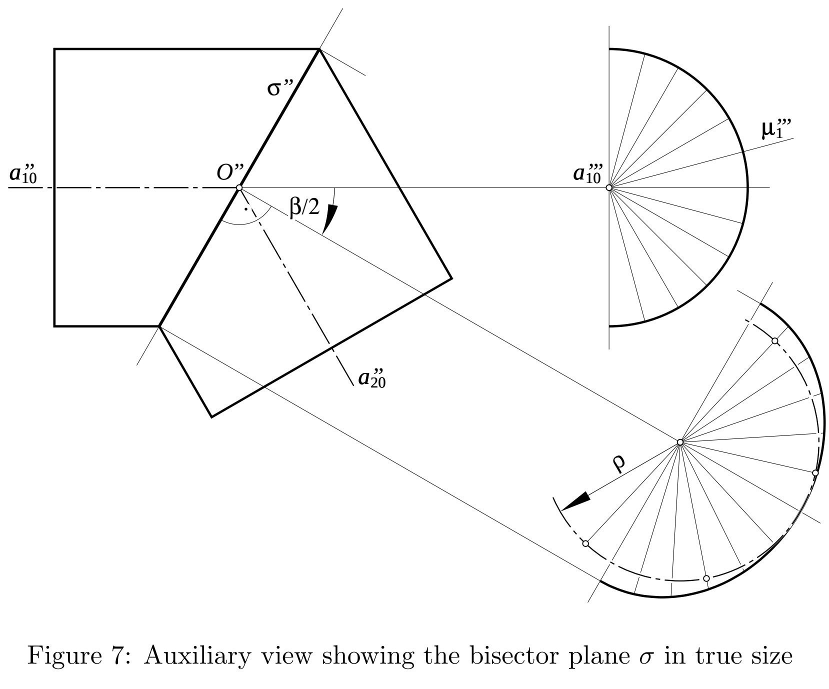

Figure 7: Auxiliary view showing the bisector plane σ in true size

One has to note that the centers of the steel balls do not run with constant speed on their circular path in the bisector plane σ. The auxiliary view in Fig. 7 reveals that the regular spacing shown in the side view is affinely distorted in σ. The distances between any two spheres vary during the motion. This results in the fact that the openings in the ball bearing Σ_3 must have a longitudinal shape in order to permit this variation of mutual distances. (This can be also be observed in the top view of Fig. 5). It is an easy but interesting exercise to compute the maximal and minimal distance which can appear between adjacent steel ball centers in dependence of the angle β made by a_{10} and a_{20}.

REFERENCES

1. STACHEL, H., "Exercises in Applied Descriptive Geometry", Proc. 2nd ICECG, Beijing (China), 200–205, 1984.

2. FUHS, W. and STACHEL, H., "Applied Descriptive Geometry Problems",
Proc. 3rd ICEEGDG, Vienna, vol. 1, 165–168, 1988.