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KINEMATIC ANALYSIS OF A HIGHER-PAIR MECHANISM FOR THE GENERATION OF INVOLUTE TOOTH PROFILES

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ABSTRACT

The planar motion between two rigid bodies can be represented by a pair of centrodes, rolling with respect to each other, without sliding, thereby defining the successive positions of the instant center of rotation (IC) of their relative motion. A pair of conjugate profiles can be assumed coincident with these centrodes, in order to obtain pure-rolling profiles, as in the case of friction wheels, thereby providing a maximum efficiency, but a low force transmission. A different pair of conjugate profiles is usually chosen, as close as possible to the IC, in order to obtain a low sliding velocity and, hence, high efficiency, along with an acceptable force transmission under a low pressure angle. Therefore, when the planar motion is assigned by a pair of centrodes, the Camus theorem allows the kinematic synthesis of a pair of conjugate profiles upon choosing a generic smooth curve as auxiliary centrode, which must be tangent to both centrodes at the IC, as first proposed and applied by Camus and Reuleaux [1,2]. Thus, both conjugate profiles can be obtained upon choosing a suitable tracing point or an enveloping smooth curve, as attached to the auxiliary centrode, then generating the ensuing pure rolling motion on the assigned centrodes upon prescribing the required profiles as either trajectories or envelope curves, respectively.

In the case of involute planar gears with a constant transmission ratio, the centrodes are represented by the pitch circles, the conjugate profiles by the involute curves of the base circles, for a given pressure angle that is usually the standard value of 20°; these circles represent the loci of the centers of curvature of the involute tooth profiles, i.e., the evolute curves. The Camus theorem is applied upon choosing as auxiliary centrode the tangent line to the pitch circles at the IC; then, a second line is attached to the previous one as an enveloping smooth curve, thus obtaining the conjugate tooth profiles as involute curves of the base circles. An algorithm based on this method, was formulated and validated by the authors [3]. This result can be also obtained upon application of the Camus theorem, while choosing a logarithmic spiral as auxiliary centrode, thus generating both conjugate involute tooth profiles as trajectories of the same tracing point. In fact, during the pure-rolling motion of the logarithmic spiral on the pitch circle of a gear, the involute of the base circle is traced by its pole or asymptotic point. The involute straight tooth profiles of the rack can be generated in the same way. First results on the application of this method to the synthesis of involute tooth profiles for both the planar and the spherical cases were proposed earlier [4-6].

The subject of this paper is the kinematic analysis of a novel higher-pair mechanism with one d.o.f. for involute-tooth profile generation, developed as per Fig. 1, which sketches the pure-rolling engagement of the pitch circle $\mathcal{P}$ of the involute driving gear 2 and the logarithmic spiral $\mathcal{S}$ that plays the role of the pitch curve of the driven non-circular gear 3, member 1 being the fixed frame. In particular, gear 2 rotates about the fixed revolute joint of center O_2, while gear 3 undergoes general planar motion with respect to 1, the pertinent IC being represented by the contact point I.

When the logarithmic spiral pitch curve $\mathcal{S}$ of gear 3 rolls on the pitch circle $\mathcal{P}$ of gear 2 upon performing the relative motion of 3 on 2, the evolute curve $\mathcal{E}$ of $\mathcal{S}$ rolls on line 1, which plays the role of the fixed centrode consequently, $\mathcal{E}$ becomes the moving centrode of the absolute motion of 3 on 1. The logarithmic spiral evolute curve $\mathcal{E}$ of $\mathcal{S}$ is congruent to the original curve $\mathcal{S}$, simply rotated and properly scaled; they are assumed to be rigidly joined, thereby forming one single rigid body, namely, 3. The Aronhold-Kennedy theorem is satisfied, since the ICs of 2 and 3, O_2 and I, respectively, are aligned with P_0, the IC for the relative motion of 3 w.r.t. 2. The involute tooth profile $\mathcal{I}_V$ of gear 2 is traced by the pole or asymptotic point O of both logarithmic spiral curves $\mathcal{S}$ and $\mathcal{E}$ of the non-circular gear 3, during the relative pure-rolling motion of $\mathcal{S}$ on the pitch circle $\mathcal{P}$, shown in Fig. 1 as a dotted line. $\mathcal{I}_V$ is also the involute curve of the base circle $\mathcal{B}$ of radius ρ_2, r_2 and r_3 being respectively, the pitch radius of $\mathcal{P}$ and the radius of curvature of $\mathcal{S}$ at point P_0 or, similarly, the radius of the corresponding osculating circle $\mathcal{C}$.

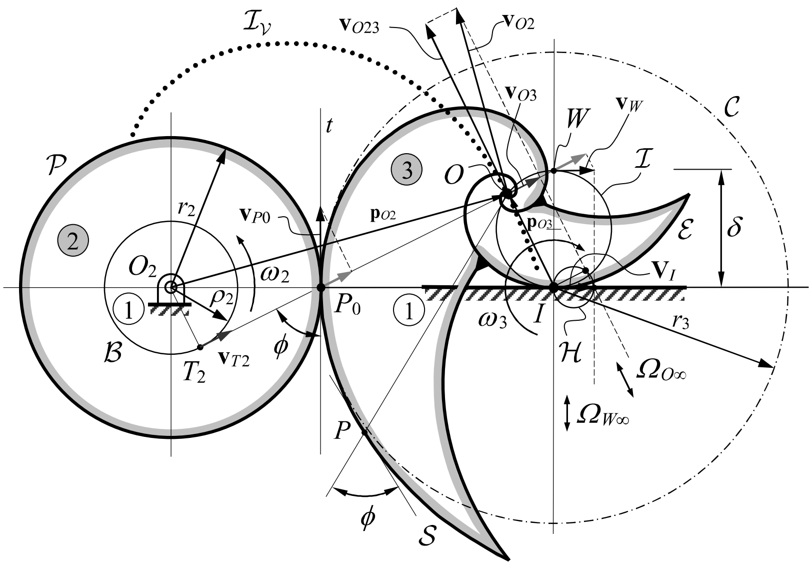

Figure 1. Higher-pair mechanism for the generation of involute tooth profiles.

During the planar motion of the proposed one-d.o.f. higher-pair mechanism of Fig .1 and, in particular, the absolute pure-rolling motion of 3 on 1, the same point O traces also the straight-line tangent to $\mathcal{B}$ at point T_2 and passing through P_0, which is also the line of action of the pair of involute-gears given by the circular gear 2 and the logarithmic non-circular gear 3. The pressure angle ϕ made by the line of action with the common tangent line t to $\mathcal{P}$ and $\mathcal{S}$ at P_0 coincides with that made by all rays stemming from the pole O of $\mathcal{S}$ and $\mathcal{E}$, with the tangent line to $\mathcal{S}$ at the corresponding intersection point P.

The kinematic analysis is developed upon assigning a constant, counterclockwise angular velocity ω_2 to the driving gear 2 and determining the velocity vector $\mathbf{v}_{P0}$ of P_0, as belonging to gears 2 and 3, respectively, thereby obtaining the instantaneous angular velocity ω_3 of the driven non-circular gear 3. Consequently, the constant velocity vectors $\mathbf{v}_{T2}$ and $\mathbf{v}_{O3}$ are determined by referring to the ICs O_2 and I respectively, and observing that they are both equal to the component of $\mathbf{v}_{P0}$ along the line of action. Moreover, the relative velocity vector $\mathbf{v}_{O32}$ of pole O during the generation of the $\mathcal{I}_V$ involute tooth profile, is given by

$$\mathbf{v}_{O32} = -\mathbf{v}_{O23} = \mathbf{v}_{O3} - \mathbf{v}_{O2} \quad (1)$$

where $\mathbf{v}_{O3} = \boldsymbol{\omega}_3 \times \mathbf{p}_{O3}$ and $\mathbf{v}_{O2} = \boldsymbol{\omega}_2 \times \mathbf{p}_{O2}$, as per Fig. 1.

A suitable algorithm for the kinematic analysis of the proposed higher-pair mechanism of Fig .1 with one d.o.f. was formulated upon application of the Camus theorem, then using the logarithmic spiral as auxiliary centrode, thus generating directly the involute tooth profiles of gears and racks. Moreover, the general planar motion of gear 3 is analyzed in depth by means of the fundamentals of kinematics, while including the determination of the inflection circle $\mathcal{I}$ of diameter $\delta = r_3/\tan\phi$, the acceleration pole and the Hartmann circle $\mathcal{H}$ of diameter V_W, equal to the magnitude of the displacement velocity of I and the velocity magnitude of the inflection pole W, along with the cubic of stationary curvature and the Ball point, derived earlier [7] via two different means, based on the application of Bottema's invariants and Cesàro's intrinsic geometry, respectively. The proposed algorithm was implemented in Matlab and validated by means of numerical and graphical results. Moreover, it is noteworthy that the proposed one-d.o.f. higher-pair mechanism can be also applied to the generation of involute tooth profiles of non-circular gears with convex pitch curves, then extended to involute bevel gears. The main advantage of the procedure reported here is the use of the pitch circles or curves, and cones, thus obviating the application of envelope theory.

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