Descriptive Geometry –
Vision Guided Spatial Reasoning

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Heritage and Expectations in Research and Teaching. 1. History and Epistemology

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0. Introduction

The aim of my presentation is to explain the role of Descriptive Geometry education.

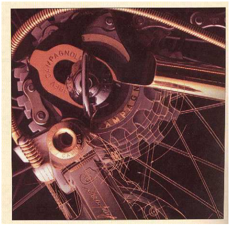
In the *hierarchy of sciences* Descriptive Geometry is placed *within or next* to Mathematics, but also close to Architecture, Mechanical Engineering, and Engineering Graphics.

A few examples shall highlight that Descriptive Geometry *provides a training* of the students' intellectual capability of *space perception* and *spatial reasoning.*

Descriptive Geometry is therefore *of incotestable importance for all engineers, architects, physicians, and natural scientists.*

1. What is Descriptive Geometry ?

In American textbooks Descriptive Geometry seems to be restricted to standard constructions like the determination of the true length of a line segment or the intersection of two plane polygons in 3-space.



G. B. Bertoline et al.:
Engineering Graphics Communication
Irwin Graphics Series, Chicago 1995, 900 pages

... Descriptive geometry is the mathematical foundation of engineering graphics.

Part III provides the basics of descriptive geometry, including the important concepts of true-length lines and true size and shape surfaces, and the relationships between lines and planes.

Part III also expands on the multiview drawing concepts ... Finally,

Part III introduces the essential concepts of intersections and developments

Descriptive Geometry in Europe

Browsing through German textbooks from the last five decades:

J. Krames (Vienna, 1967):

"Descriptive Geometry" is the high art of spatial reasoning and its graphic representation.

H. Brauner (Vienna, 1986) preferred the name *'Constructional Geometry'* instead of Descriptive Geometry and defined:

"Constructional Geometry" encompasses the analysis of 3D objects by means of graphical or mathematical methods applied to 2D images.

Descriptive Geometry in Europe, cntd.

F. Hohenberg (Graz, 1966) whose textbook focusses on applications of Descriptive Geometry in technology:

"Constructional Geometry" teaches how to grasp, to imagine, to design and to draw geometrical shapes.

W.-D. Klix (Dresden, 2001) gives the following extended explanation:

"Descriptive Geometry" is unique in the way how it promotes spatial reasoning, which is so fundamental for each creative activity of engineers, and how it trains the ability to express spatial ideas graphically so that they become understandable for anybody else.

my definition . . .

Definition:

Descriptive Geometry is a method to study 3D geometry through 2D images.

It provides insight into structure and metrical properties of spatial objects, processes and principles.

Typical for Descriptive Geometry is the interplay

- between the 3D situation and its 2D representation,
- between intuitive grasping and rigorous logical reasoning.

Descriptive Geometry in Europe, cntd.

Beside *projection theory*, Descriptive Geometry courses in Europe cover

- *modeling techniques* for curves, surfaces, and solids,
- insight into a broad variety of *geometric shapes* (including polyhedra),
- an intuitive approach to basic *differential geometry* (like curvature, osculating plane) of curves and surfaces,
- some *3D analytic geometry.*

An additional aim is to develop and refine the students' *problem-solving skills.*

'Learning by doing' is an important methodological principle.

G. Monge’s definition

La Géométrie descriptive a deux objets:

- **le premier**, de donner les méthodes pour représenter sur une feuille de dessin qui n’a que deux dimensions, savoir, longueur et largeur, tous les corps de la nature qui en ont trois, longueur, largeur et profondeur, pourvu néanmoins que ces corps puissent être définis rigoureusement.

- **Le second objet** est de donner la manière de reconnaître, d’aprés une description exacte, les formes des corps, et d’en déduire toutes les vérités qui résultent et de leur forme et de leurs positions respectives.

G. Monge (1746–1818)
Place de Monge, Beaune
Dep. Côte-d’Or, France

two main objectives of Descriptive Geometry

The *two main objectives* of Descriptive Geometry

- imaging
- analysing

3D objects

date back to its founder G. Monge.

The word *'drawing'* does not appear in Monge's definition.

In Descriptive Geometry *drawings* are the guide to geometry but not the main aim; we teach geometry instead of construction techniques.

Further comments on Descriptive Geometry

The great mathematician Carl Friedrich Gauss writes 1813 in his review on G. Monge's book:

"One cannot deny the advantages of the analytical method... However, it will always be very important to cultivate the geometric method, too. Apart from being often more direct and shorter than analysis, it will show its genuine elegance...."

"Especially we must praise Monge*'s 'Géométrie descriptive' for its clearness and conciseness...*

"... So we must recommend the study of this book as a substantial intellectual nourishment, capable of enliving and preserving the genuine geometric spirit, a spirit which is missed sometimes in the mathematics of our time."

Further comments on Descriptive Geometry

Only people with a profound knowledge in Descriptive Geometry are able to extended use of CAD programs.

For similar reasons the importance of mathematics is still increasing though computers take over the computational labour.

Descriptive Geometry is more than 'descriptive' geometry —

as well as *Geometry* is more than its literal sense, i.e., 'measuring the earth'.

What a Descriptive Geometry course should effect

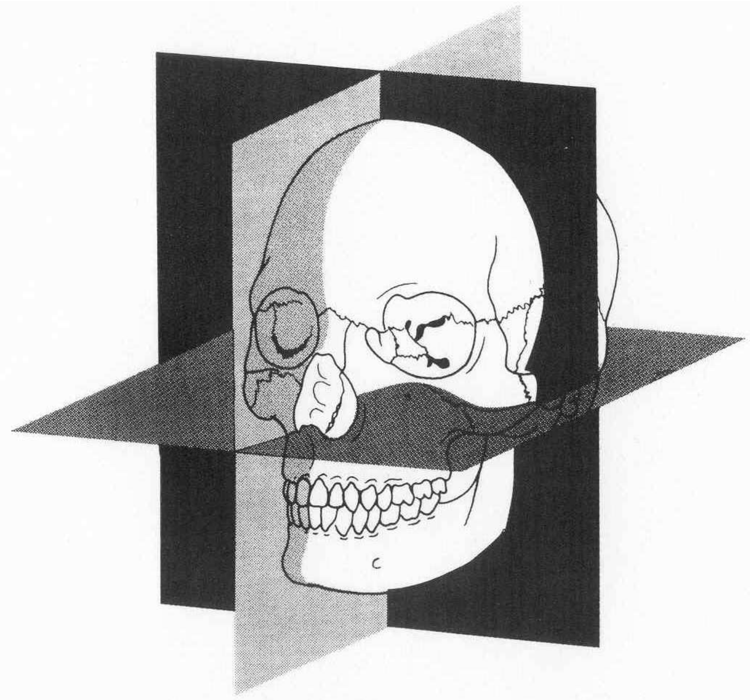
Education in Descriptive Geometry brings about the ability

- to comprehend spatial objects from given principal views (top view, front view, and side view),
- to specify and grasp particular views (auxiliary views),
- to get an idea of geometric idealization (abstraction), of the variety of geometric shapes, and of geometric reasoning.

The first two items look elementary. However, these intellectual abilities are so fundamental that many people forget how hard they were to achieve.

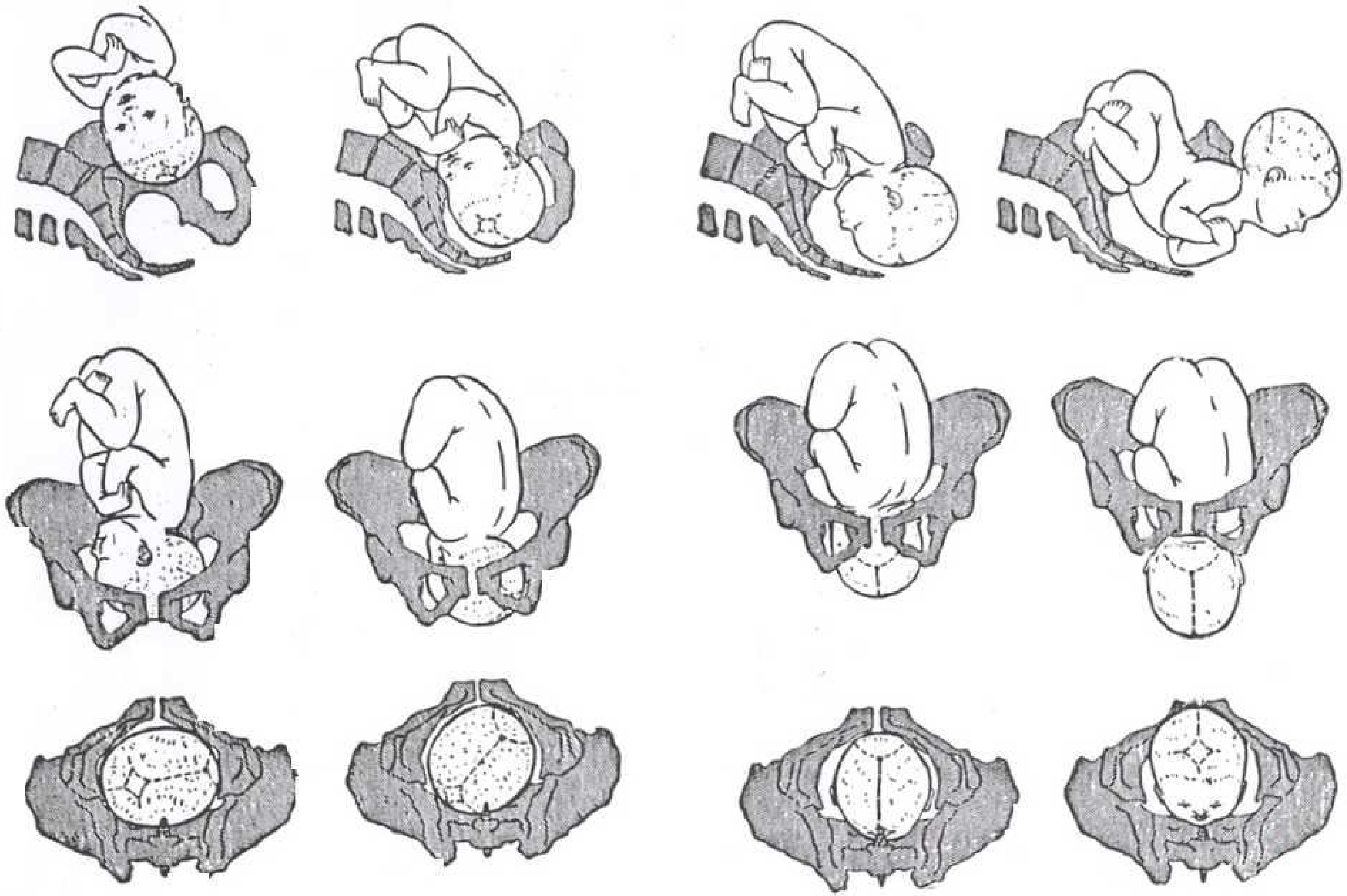
2a) The importance of principal views

- Principal views are *more or less abstract*. But abstraction often simplifies.
- Inspecting these plane views is much easier than to concentrate on the true 3D structure or process.
- It needs *training* to become familiar with this kind of representation.
- *Medical doctors* often hold in esteem their Descriptive Geometry education.



Explainig the principal views to dentists

Principal views for gynaecologists



Courtesy: Prof. Albert Schmid-Kirsch, Leibniz Universität Hannover

2b) The art of specifying appropriate views

For a detailed 3D analysis particular views (auxiliary views) often reveal the spatial situation.

Views showing planes in edge view or lines in point view can be the key for the solution of a 3D problem.

Appropriate views make the sublime art of Descriptive Geometry.

Only in such courses students are trained to specify and to grasp such views.

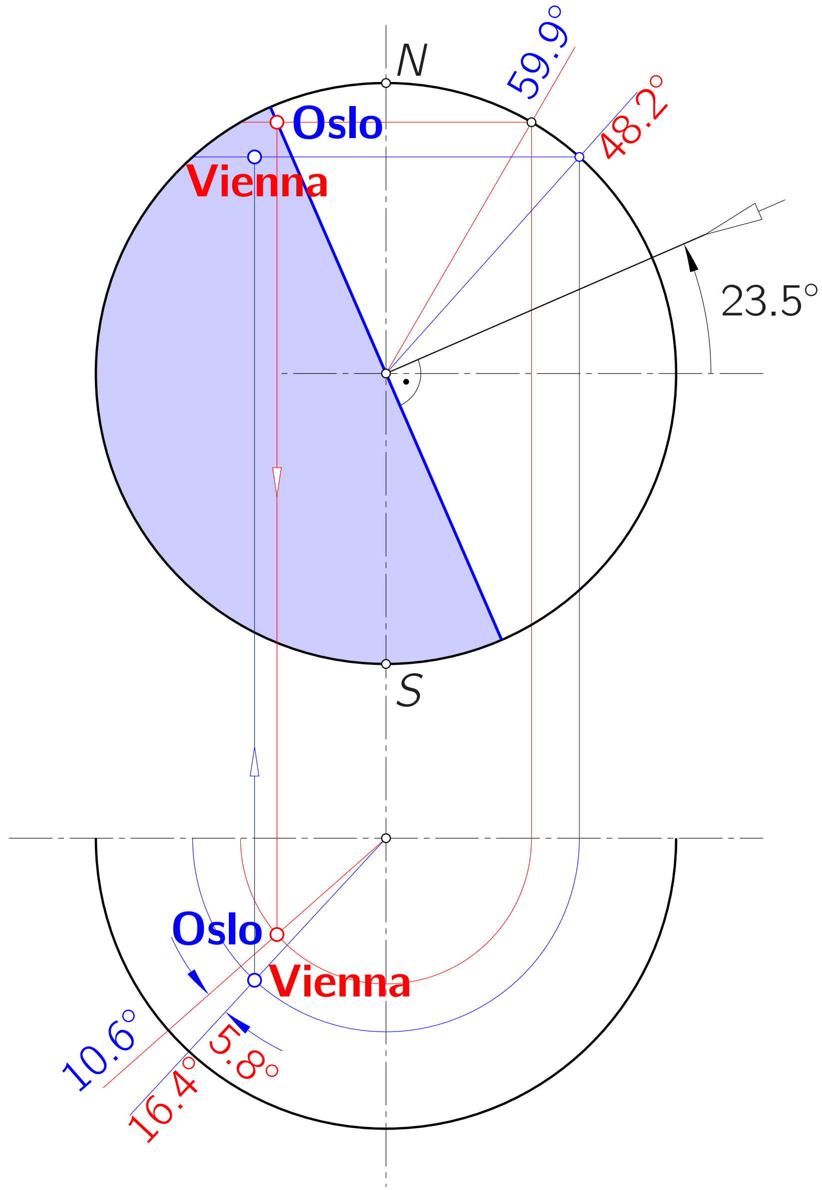
Two examples

Where does the sun rise earlier on June 21, in Oslo or in Vienna?

city	*Eastern longitude*	*Northern latitude*
Oslo	10.6°	59.9°
Vienna	16.4°	48.2°

We specify a front view with sun rays parallel to the image plane.

We seek Vienna relatively to Oslo.

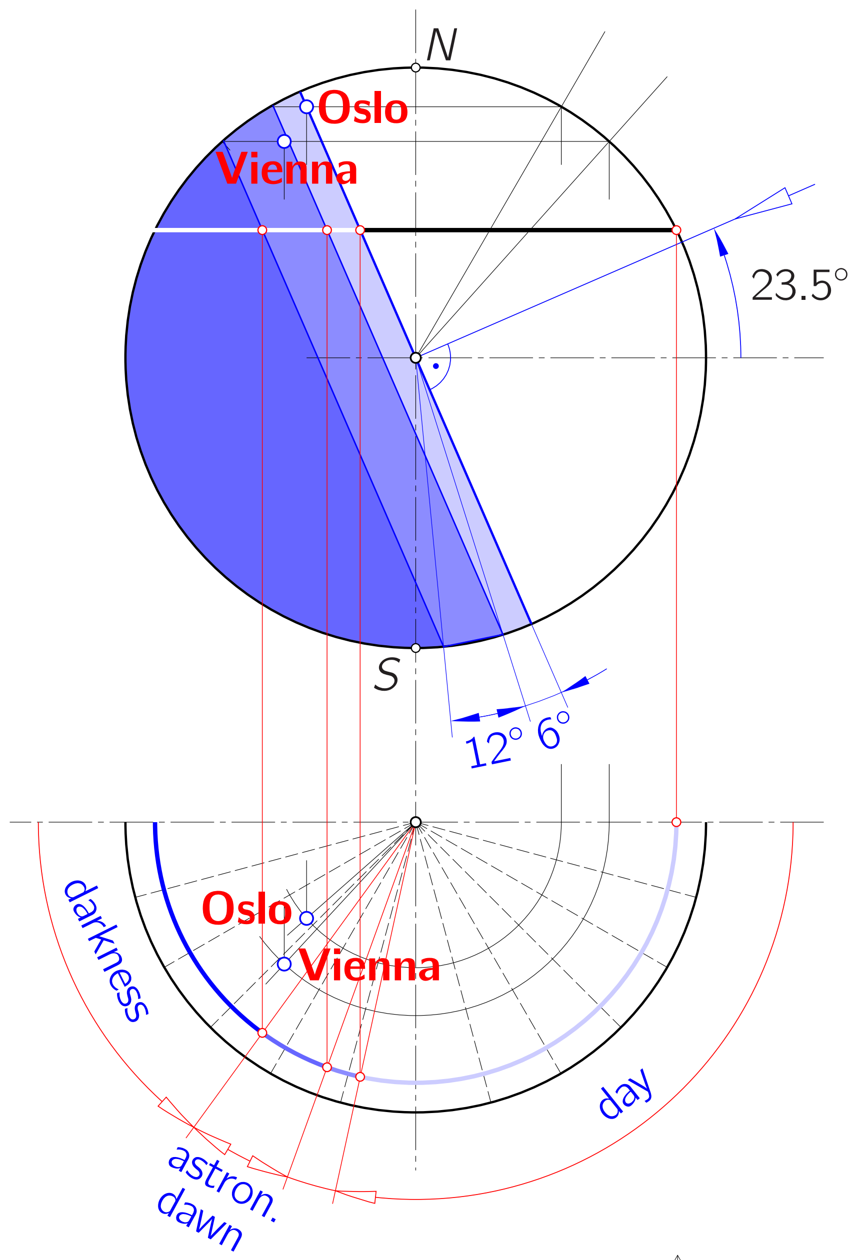


additional questions

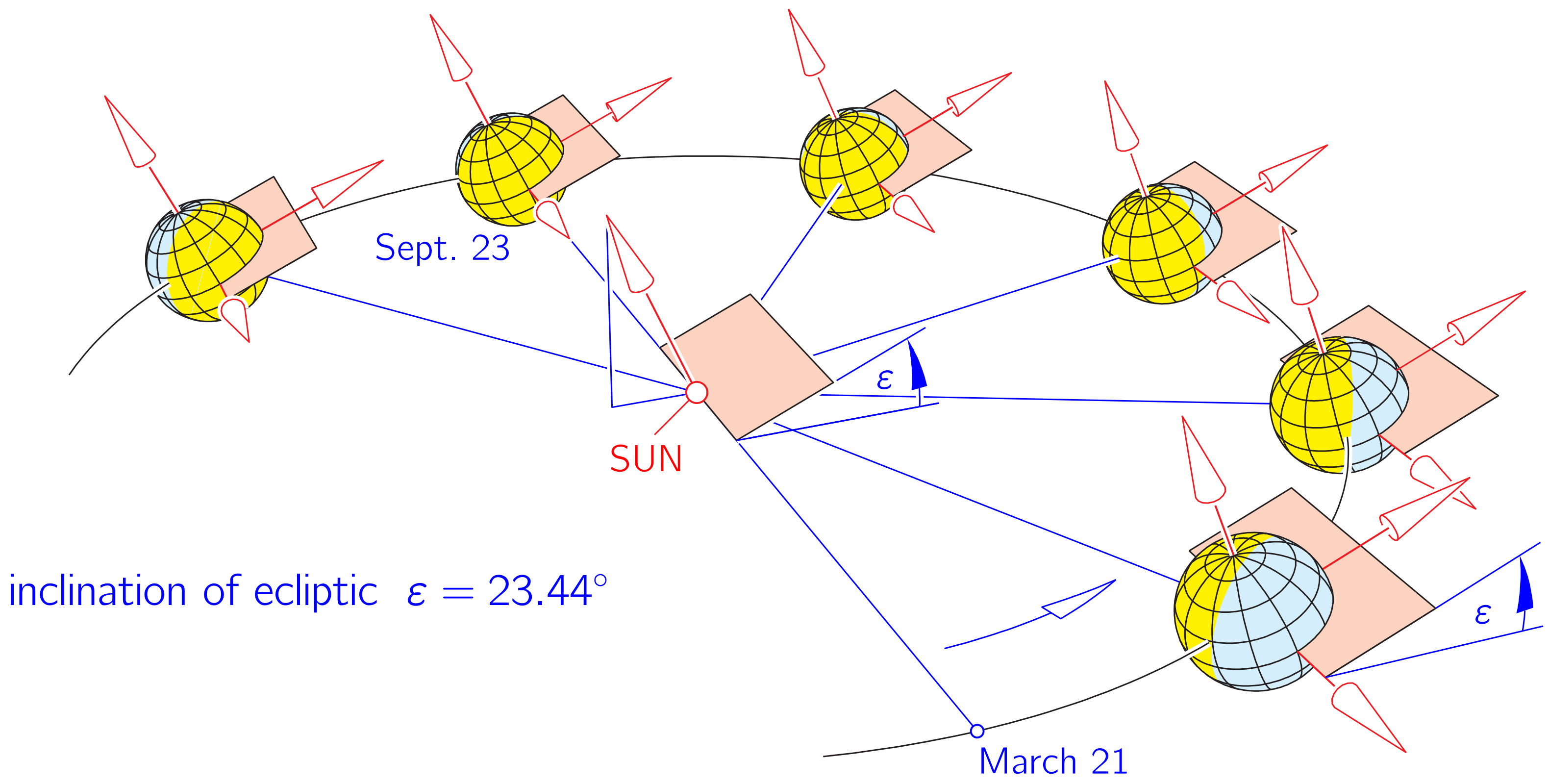
a) We can increase the precision by paying attention to the refraction in the atmosphere: For an observer the sun seems to rise while it is still approx. $0,6^\circ$ under the local horizon.

b) In the zone of astronomic dawn the sun is between 6° and 18° under the local horizon.

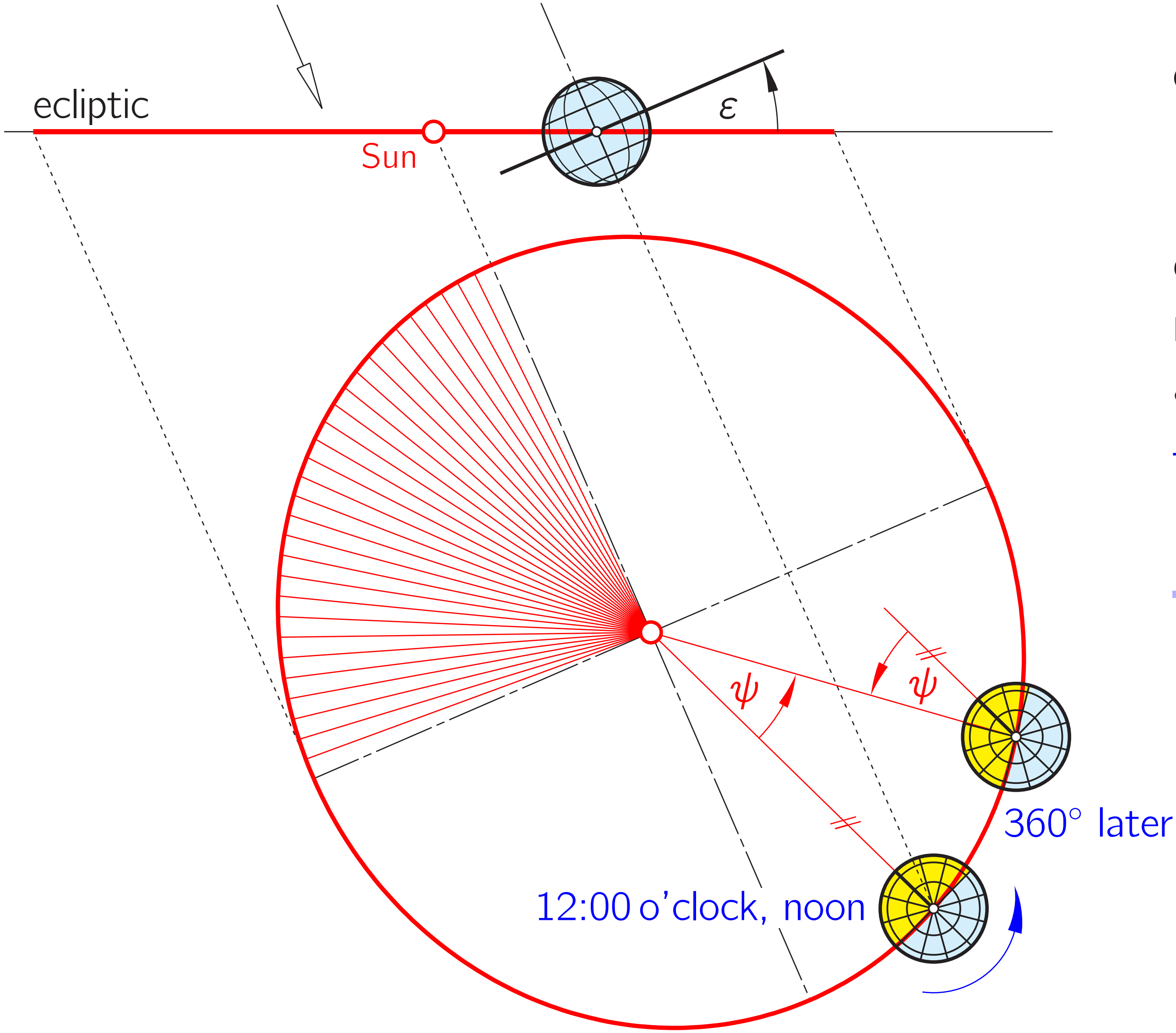
c) By inspection we observe that the period of dawn is shorter in the neighborhood of the equator.



The difference between median time and true time



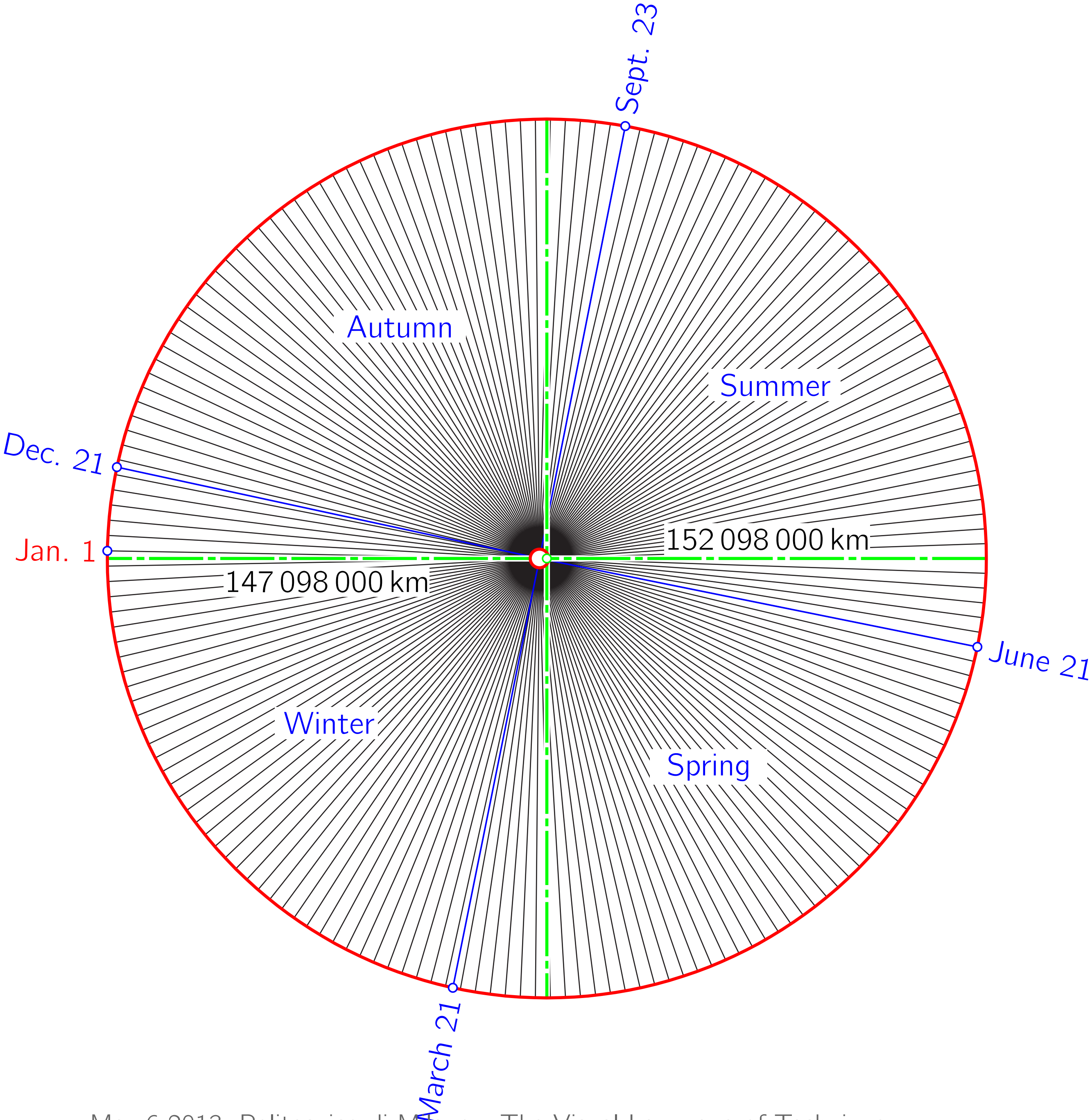
During the movement of the earth around the sun the earths's axis preserves its direction (in first order approximation). ε is the obliquity of the ecliptic.



The auxiliary view in direction of the earth's axis reveals:

During consecutive sun culminations the earth must rotate about its axis through an angle of $360° + \psi \Longrightarrow$ the duration of days (apparent sun time) varies!

Even for a circular path of the earth the center angle ψ for 24 hours (mean time) varies — because of the affine distortion.



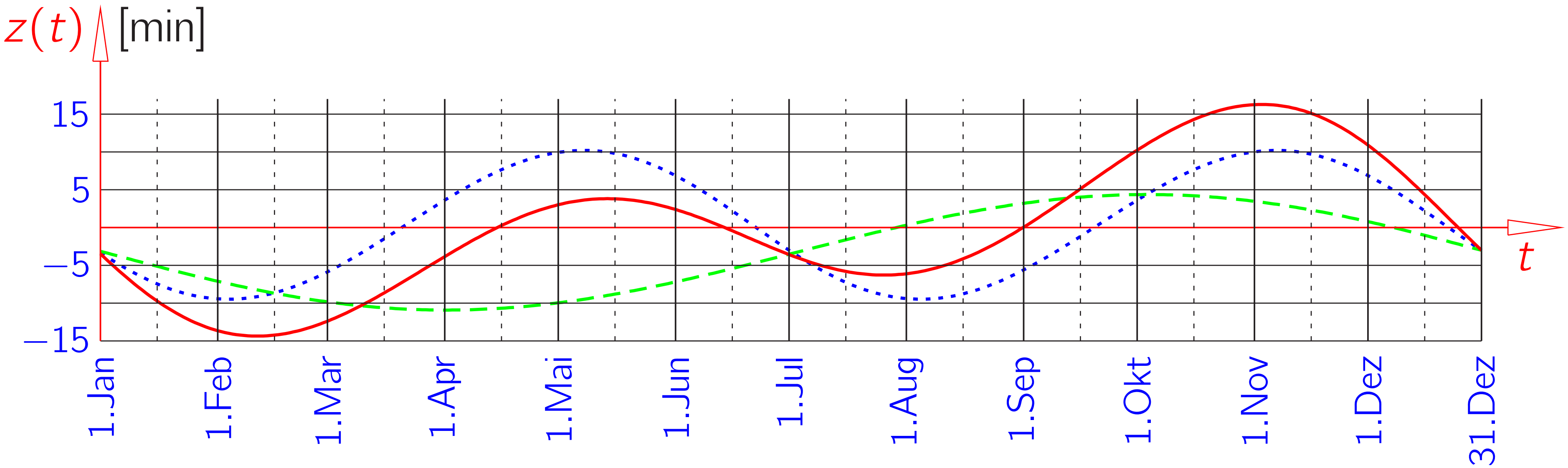
Moreover, the earth runs around the sun along an ellipse with constant areal velocity (Kepler's First Law).

interval of subdivision 2 days (24 hours mean time)

Data:

$$a = 149.598\,10^6\,\text{km}$$
$$b = 149.577\,10^6\,\text{km}$$
$$e = 2.500\,10^6\,\text{km}$$
$$\varepsilon = e/a = 0.01671$$

The difference between median time and true time



Equation of time: true time = apparent solar time = mean solar time + $z(t)$

· · · · · · · · · Equation of time for a circular path $(e = 0)$

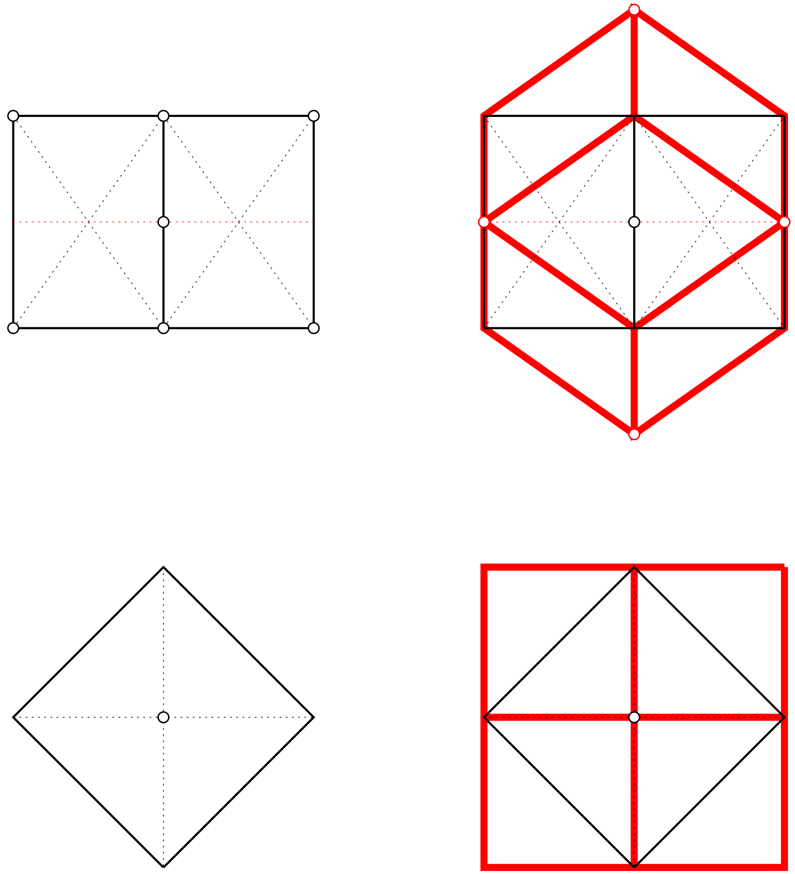
- - - - - - - Equation of time in the elliptic case with $\varepsilon = 0$

Mentally manipulating 3D objects?

The *rhombic dodecahedron* can be built by erecting quadratic pyramides with 45° inclined planes over each face of a cube.

Any two coplanar triangles can be glued together forming a rhomb.

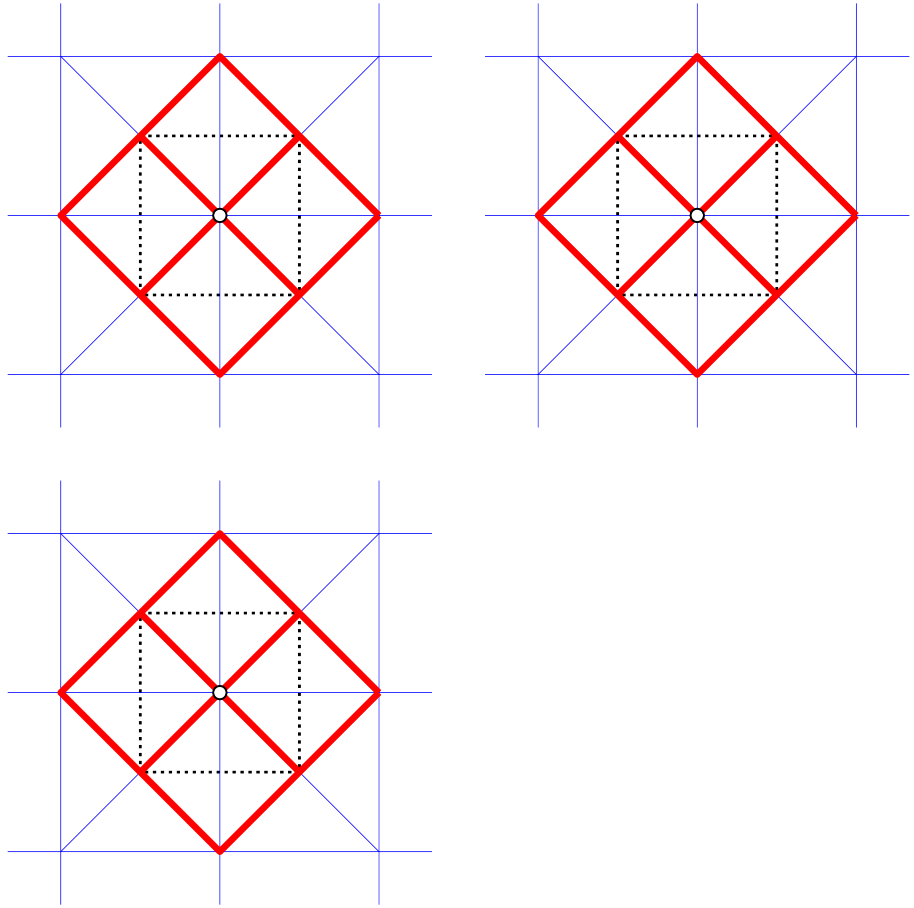
Question:
How does this polyhedron look like from above when it is resting with one face on a table?



Cube and rhombic dodecahedron

Special views reveal 3D properties

The rhombic dodecahedron is the *intersection of three quadratic prisms* with pairwise orthogonal axes.

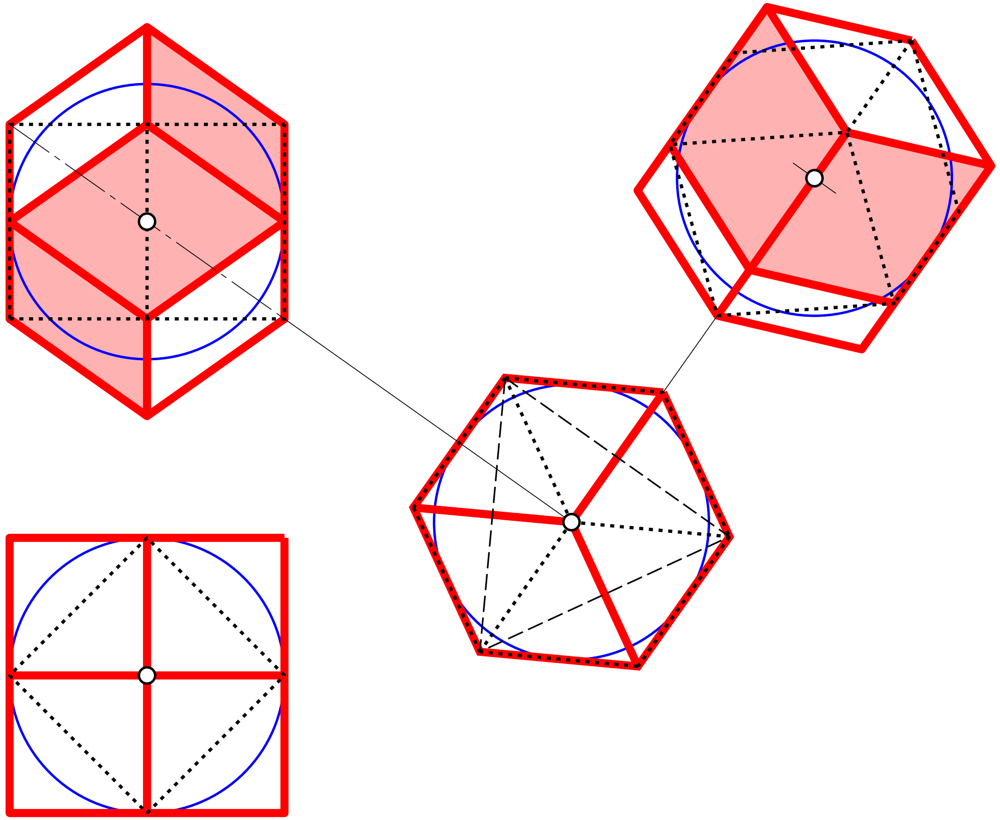


Special views reveal 3D properties

The rhombic dodecahedron is the *intersection of three hexagonal prisms* with axes in direction of the cube-diagonals.

The side and back walls of a *honey comb* belong to a rhombic dodecahedron.

Each *dihedral angle* makes 120°, and there is an in-sphere (contacting all faces of the initial cube).



A simple 3D operation carried out mentally

The rhombic dodecahedron is a space-filling polyhedron.

Proof:

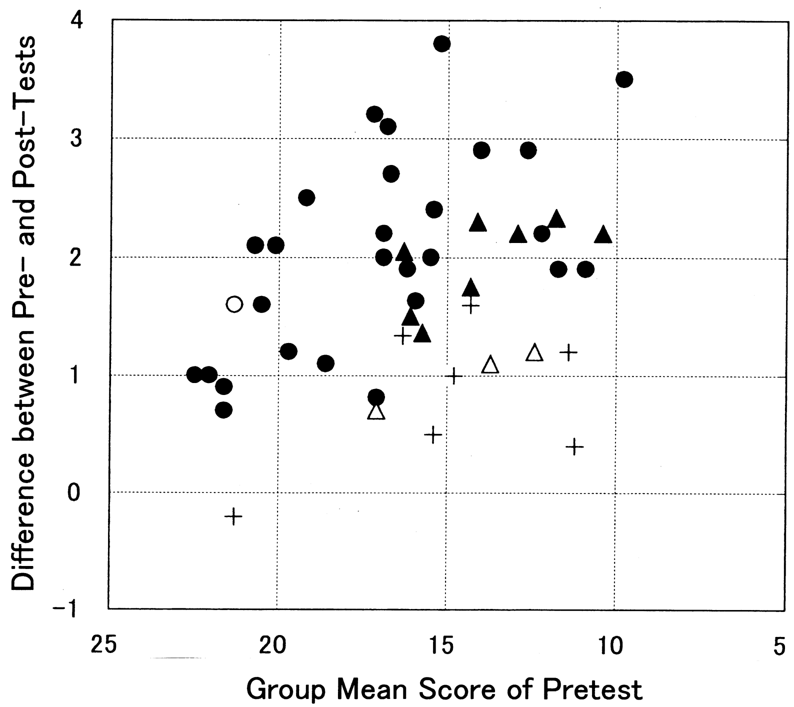
- Start with a *'3D-chessboard'* built from black and white cubes.
- Then the 'white' cubes can be *partitioned* into 6 quadratic pyramides with the vertex at the cube's center.
- Glue each pyramide to the *adjacent* 'black' cube thus enlarging it to a rhombic dodecahedron. □

Descriptive Geometry improves spatial ability

K. Suzuki:
J. Geometry Graphics **6**, 221–229 (2002)

MCT-Test: Differences between pre- and post-test at Japanese students before and after the geometry education:

- ● Descriptive Geometry
- ○ Computergraphics,
- ▲ Engineering Graphics,
- △ 3D-CAD,
- + control persons



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