Katedra matematiky, Pedagogická fakulta,
Jihočeská univerzita v Českých Budějovicích
Česká společnost pro geometrii a grafiku
Jednoty českých matematiků a fyziků

Sborník příspěvků

34. KONFERENCE O GEOMETRII A GRAFICE

Vlachovice/Sykovec, Nové Město na Moravě,

15.–18. září 2014

Proceedings of the

34TH CONFERENCE ON GEOMETRY AND GRAPHICS

Vlachovice/Sykovec, Nové Město na Moravě,

September 15–18, 2014

Department of Mathematics, Faculty of Education,
University of South Bohemia in České Budějovice
Czech Society for Geometry and Graphics
of the Union of Czech Mathematicians and Physicists

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Jihočeská univerzita v Českých Budějovicích
Branišovská 1645/31a, 370 05 České Budějovice

1. vydání

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ISBN 978-80-7394-470-4

The design of the sun-reflection-dial in Heiligenkreuz/Austria

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Extended Abstract

A few years ago, the author was involved in the design of a monument in the area of the Heiligenkreuz monastery[1]. This monument (see Fig. 2, left) should consist of a big mosaic-work (8 x 3.5 m) on a cylindrical wall Ψ and − at the center of the cylinder − of an 8 m high slim tower, called 'Gnomon', in form of a three-sided pyramid made from reflecting steel. According to the design of the French artist Philippe Lejeune (*1924), the shadow of this tower falling onto the cylindrical wall should be utilized for a time telling device.

Figure 1. Traditional sun-dial in Třeboň

We expect from a traditional sun-dial (note Fig. 1) that − independent from the season − at any given day-time t (e.g., 10 a.m.) the shadow of the style falls onto the same line. These so-called *hour-lines* are usually marked on the sun-dial. Of course, this requirement has consequences for the position of the *gnomon*, i.e., the style which casts the shadow.

[1] a Cistercian monastery, approx. 25 km south-west of Vienna, the eldest continuously occupied Cistercian monastery in the world.

Lemma 1: *The shadow casted from a style at given local time t is for each t placed on the same hour-line independently from the season if and only if the style is parallel to the earth's axis.*

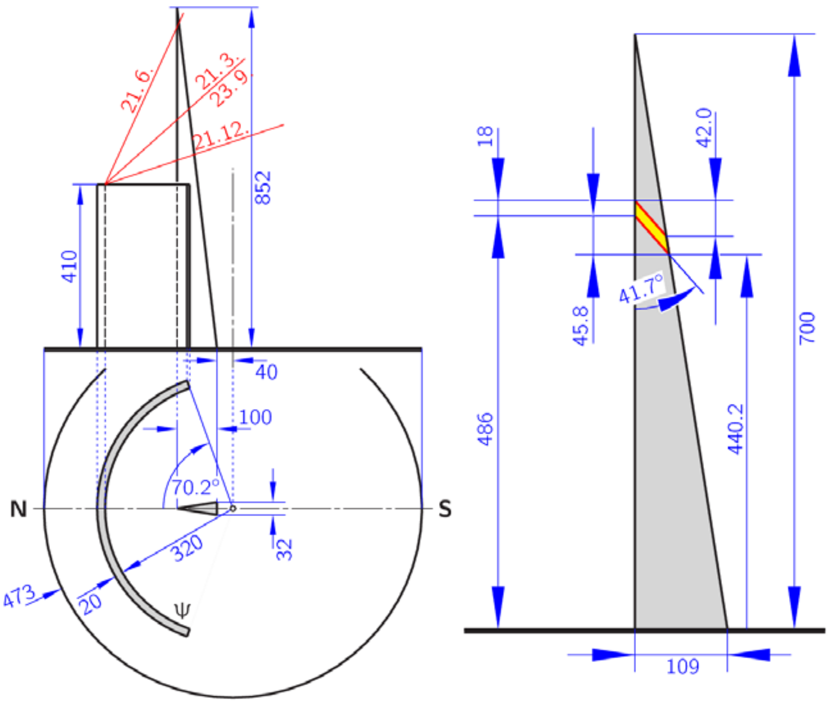

Figure 2. Left: The initial plan of the monument; Right: the final position of the reflecting stripes on the East- and Westface of the 'Gnomon'

Before we start proving Lemma 1, we insert two remarks:

1) For studying sun-dials, it is useful to adopt the geocentric view, i.e., we consider the earth to be fixed while the sun is moving relative to the earth. This motion is exactly inverse to the composition of the rotation of the earth about its axis and the translational movement along an elliptic path (ecliptic) around the sun (Fig. 3).

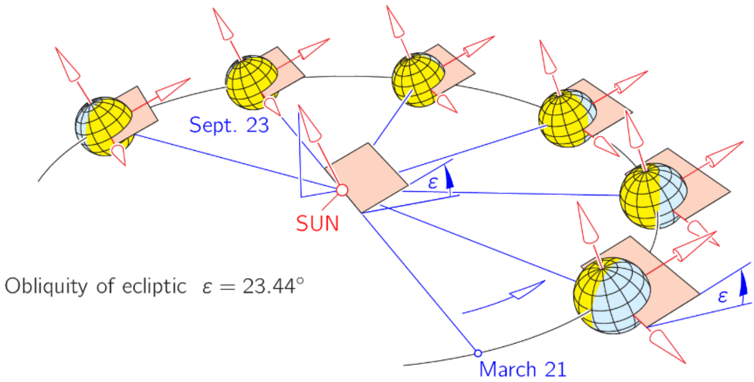

Figure 3. The earth travels around the sun along the ecliptic

2) We need to distinguish between true local time and mean time: For any position X on the earth, at noon in *true local time* the sun reaches its daily culmination relative to X, i.e., the sun passes the meridian plane of X (= plane connecting X with the axis of the earth). One day in true local time equals the period between consecutive noons. On the other hand, one day in mean time, i.e., the time we see on our clocks, equals the 365.2563...th part of one (sidereal) year which is the time the earth needs for surrounding the sun.
There is a deviation between true local time and mean time, represented in the *'Equation of Time'* (see Fig. 4). This deviation lies between approx. +15 and -15 minutes. It is a consequence of two facts:
a) the 'obliquity of ecliptic', i.e., the angle between the planes of the earth's equator and the ecliptic, and
b) the path of the earth around the sun is an ellipse which is traced with constant areal velocity (Kepler's First and Second Law).
Both facts, a) and b), imply that the duration of a single day with respect to ('w.r.t.' in short) local time varies over the year, when measured in mean time (see Fig. 4). For details see reference [4].

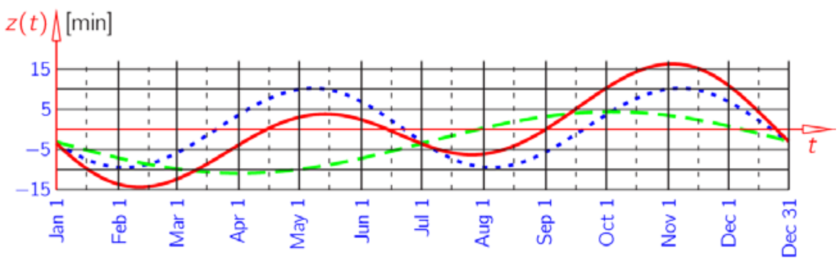

Figure 4. 'Equation of Time': z = true time – mean time. The dotted line indicates the influence of the 'obliquity of ecliptic', the dashed line shows the effect caused by Kepler's First and Second Law

Proof of Lemma 1: During the run of a year the line connecting the center *M* of the earth with the center *S* of the sun changes its inclination w.r.t. the equator plane. When the true local day-time t is kept fixed over the year, the connecting lines *SM* vary within a plane through the earth's axis. Now we replace the center *M* by an arbitrary point *P* on the earth. Therefore we apply the translation $M \mapsto P$ and conclude: All sun rays which pass every day at the same true local time *t* through point *P belong to the hour-plane* ε_t *of P*. Since ε_t is parallel to the original meridian plane, every hour-plane ε_t contains the line a parallel to the earth's axis and passing through *P*.
The style must be chosen in such a way that the shadows of all its points fall onto the same hour-line. Therefore the style must be located in the same hour-plane ε_t, and this must hold for all *t*. But this is valid only for a style along line *a* (compare [1, 2, 3]). □

With respect to the initial design for Heiligenkreuz, a vertical tower serving as a gnomon contradicts Lemma 1. However, inclining this tower to approx. 48° — the geographic latitude of Heiligenkreuz — into a pose parallel to the earth's axis would totally destroy the optical appearance of the artist's design. Therefore we started to pay attention to the *reflecting properties* of the Gnomon's faces.

Suppose, the sunbeam which is reflected at any point *Q* of the reflecting face μ meets a given surface Ψ at the point *Q'*. Then due to Fig. 5 (left), the luminous point *Q'* coincides with the shadow of point *Q* w.r.t. a *virtual sun* $\overline{S}$, the mirror of the original sun *S* w.r.t. μ. By applying this reflection to the statement of Lemma 1, we obtain:

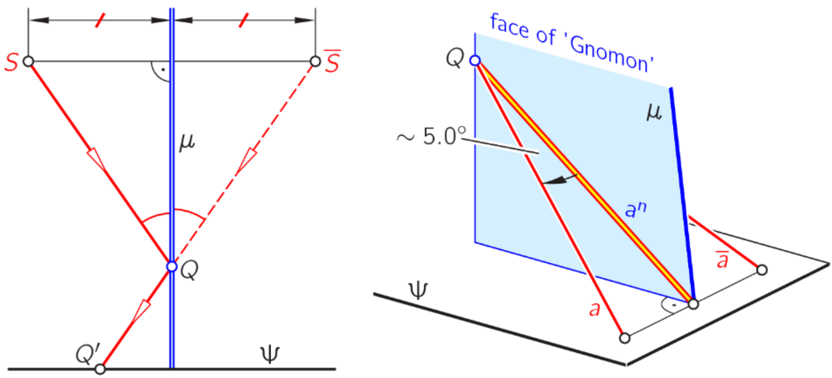

Figure 5. The luminous point *Q'* caused by reflection of a sunbeam in the plane μ at point *Q* equals the 'shadow' of *Q* w.r.t. the reflected sun

Lemma 2: *Suppose, the reflection of sunbeams along a line segment ℓ in the reflecting plane μ generates at given local time t on a surface Ψ a luminous curve segment ℓ_t. This spot ℓ_t follows for each t a 'hour-line' all over the year if and only if the reflecting segment ℓ is parallel to the mirror of the earth's axis w.r.t. μ. Because of $\ell \subset \mu$ the reflecting plane μ must be parallel to the earth's axis.*

Since no face of the 'Gnomon' in Heiligenkreuz is parallel to the earth's axis, we have to confine ourselves to an approximation: We replace $\bar{a}$ by its orthogonal projection a^n in μ, which at the same time is the orthogonal projection of line *a* (Fig. 5, right). This is admissible as Fig. 6 reveals.

Figure 7 shows how the monument 'Epiphanie' with the hour-lines in the final status looks like. The photo was taken on August 14 and shows in the upper right corner the (additionally marked) luminous stripe. It indicates approx. 8:05 a.m. However, we must pay attention to the European summertime. And, in addition, for midth of August the Equation of Time (Fig. 4) shows 5 minutes delay of the sun against CET (Central European Time). Hence, the result is approx. 9:10 a.m. which comes very close to the date stored in the camera.

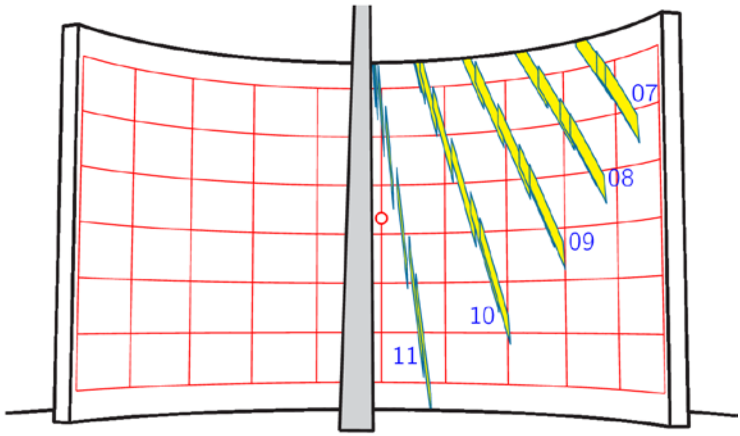

Figure 6. The luminous stripes ℓ_t on the wall Ψ vary from month to month, but can be combined to hour-lines

The photo in Fig. 7 shows near the bottom some irregular light concentrations which can be confusing for visitors. These strange looking spots have nothing to do with the sun-dial itself; they are caused by slight bendings of the lower parts of the East- and West-face due to production errors.

Figure 7. The final appearance of ’Epiphanie’, a declared monument „for Freedom of Conscience and Religion as a Foundation for Peace“

References

[1] Glaeser G., Geometry and its Applications in Arts, Nature and Technology, Springer Wien, New York, 2012.

[2] Glaeser G., Hofmann W., About minute-precise sundials for mean time, IBDG (Informationsblätter der Geometrie) 23/2, 40–46, 2004.

[3] Hohenberg, F., Konstruktive Geometrie in der Technik, 3. Aufl., Springer-Verlag, Wien, 1966.

[4] Stachel H., The Design of the New Sun-Reflection-Dial in Heiligenkreuz/ Austria, Proceedings moNGeometrija 2014, Vlasina/Serbia, 285–296.