The design of the new sun-reflection-dial in Heiligenkreuz

Hellmuth Stachel

TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

stachel@dmg.tuwien.ac.at — http://www.geometrie.tuwien.ac.at/stachel

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1. The artist's original design

Cistercian monastery Heiligenkreuz, with the marked monument **'Epiphanie'**

"for Freedom of Conscience and Religion as a Foundation for Peace"

1. The artist's original design

. . . by the way, the monastery is also famous

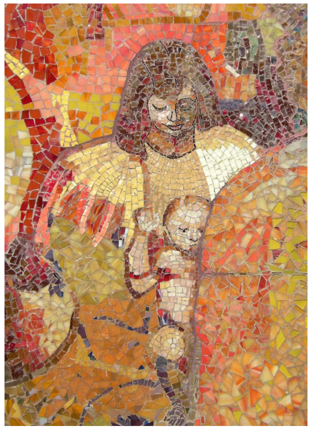
- for its 'singing monks' and
- for its Theological Faculty.

‘Epiphanie’, designed by the French artist Philippe **Lejeune** (born 1924) consisting of a mosaic-work (8×3.5 m) on cylindrical wall, and a 7 m high three-sided pyramid, made from steel and called *‘Gnomon’*

1. The artist's original design

The monument was financed by "Verein Moderner Sakralbau", a private Austrian organization which promotes modern art for Christian churches.

1. The artist's original design

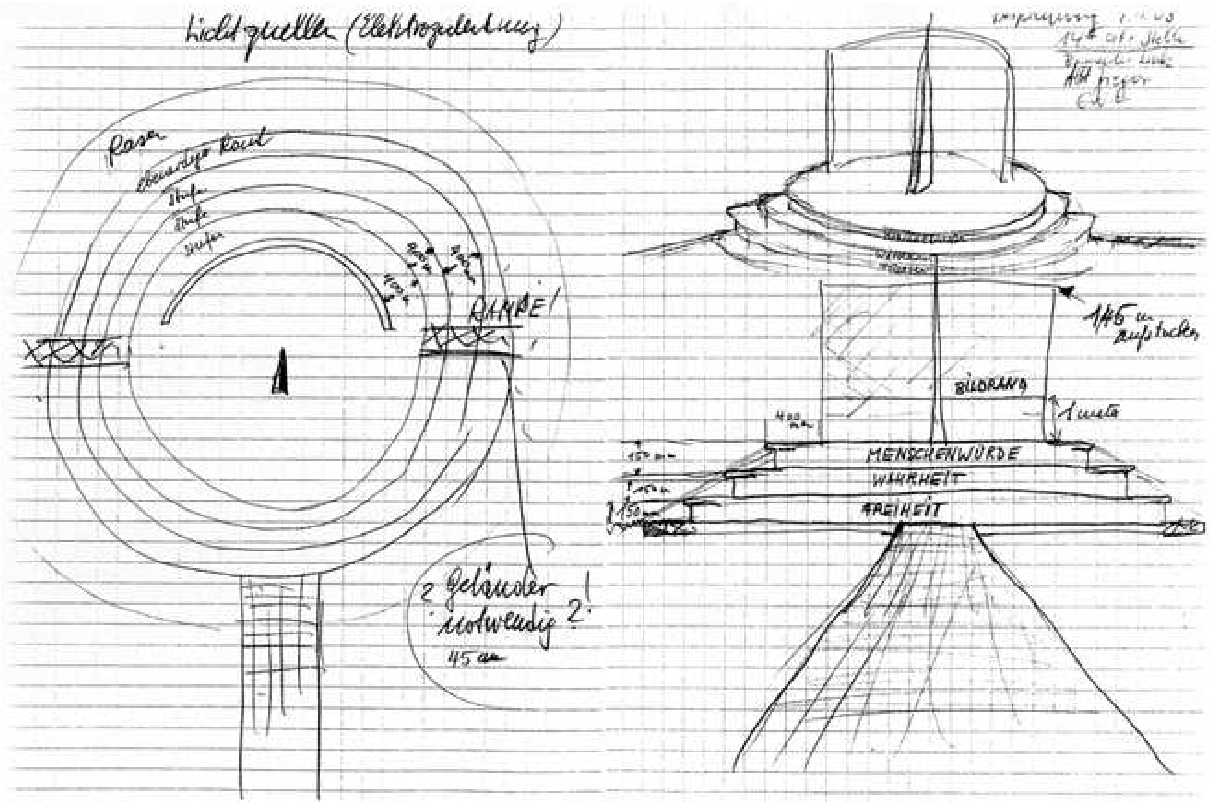


At the beginning there existed only the painting *"Epiphanie"* showing in the center the Holy Mary with her baby Jesus.

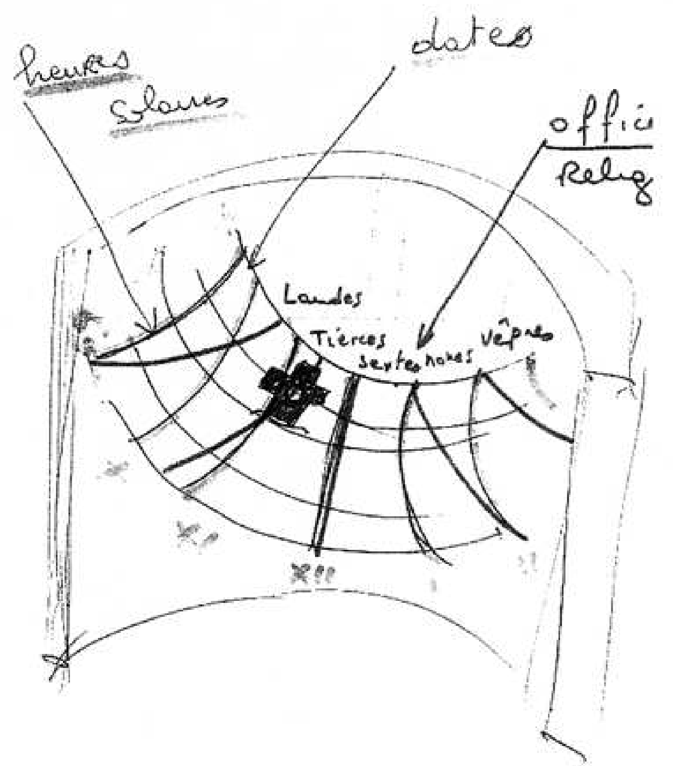
'Epiphanie' means 'the appearance of Jesus'; this event is celebrated in roman-catholic churches on January 6.

1. The artist's original design

. . . there existed sketches of the artist



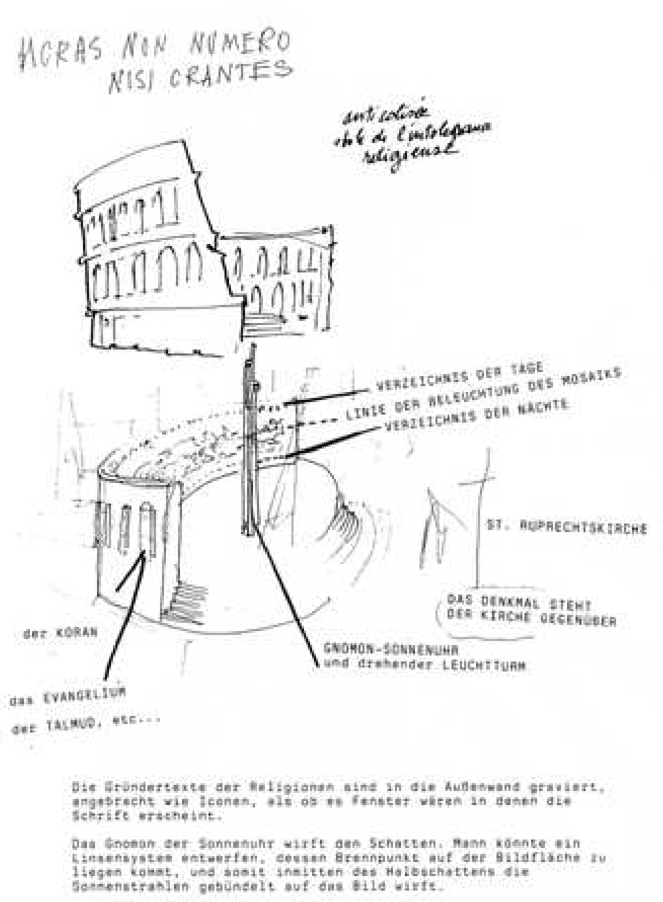
1. The artist's original design



the mosaic on the cylindrical wall should also work as a sundial;

and on January 6 a beam of sunlight should touch the depicted baby Jesus;

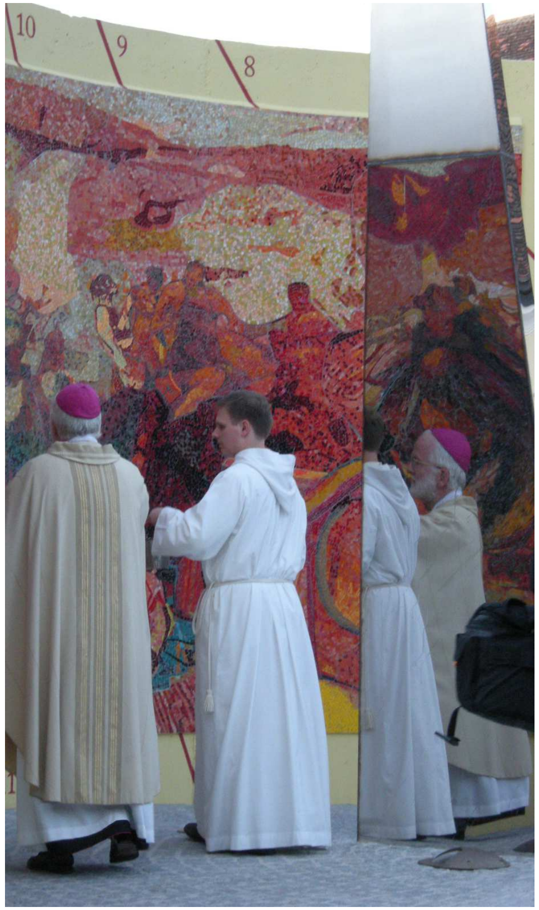
1. The artist's original design



it should cause an effect like an advertisement pillar or like an amphitheater, e. g. like the Colosseum.

The 'Gnomon' as a mirror should intensify this impression;

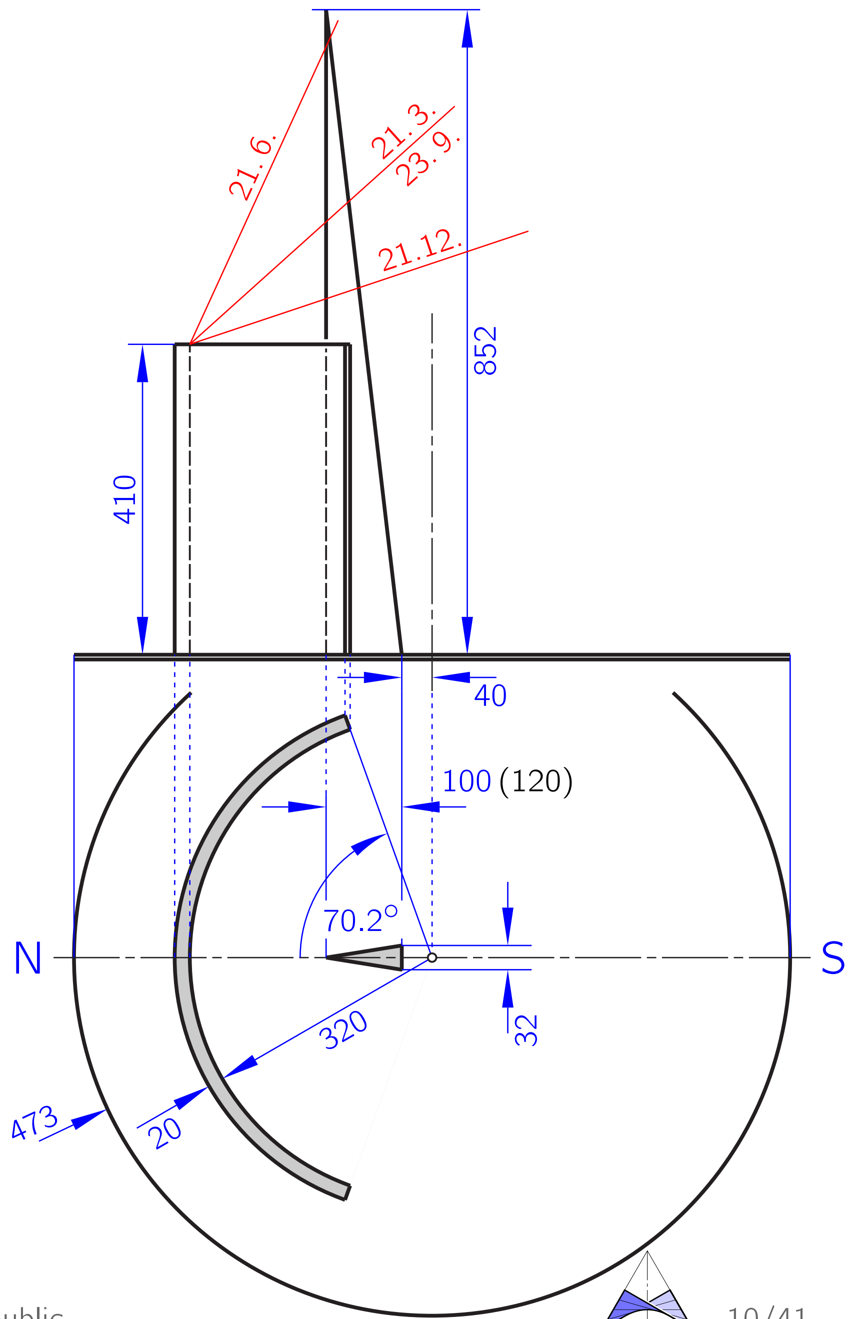
visitors should recognize themselves within the depicted crowd of visitors.



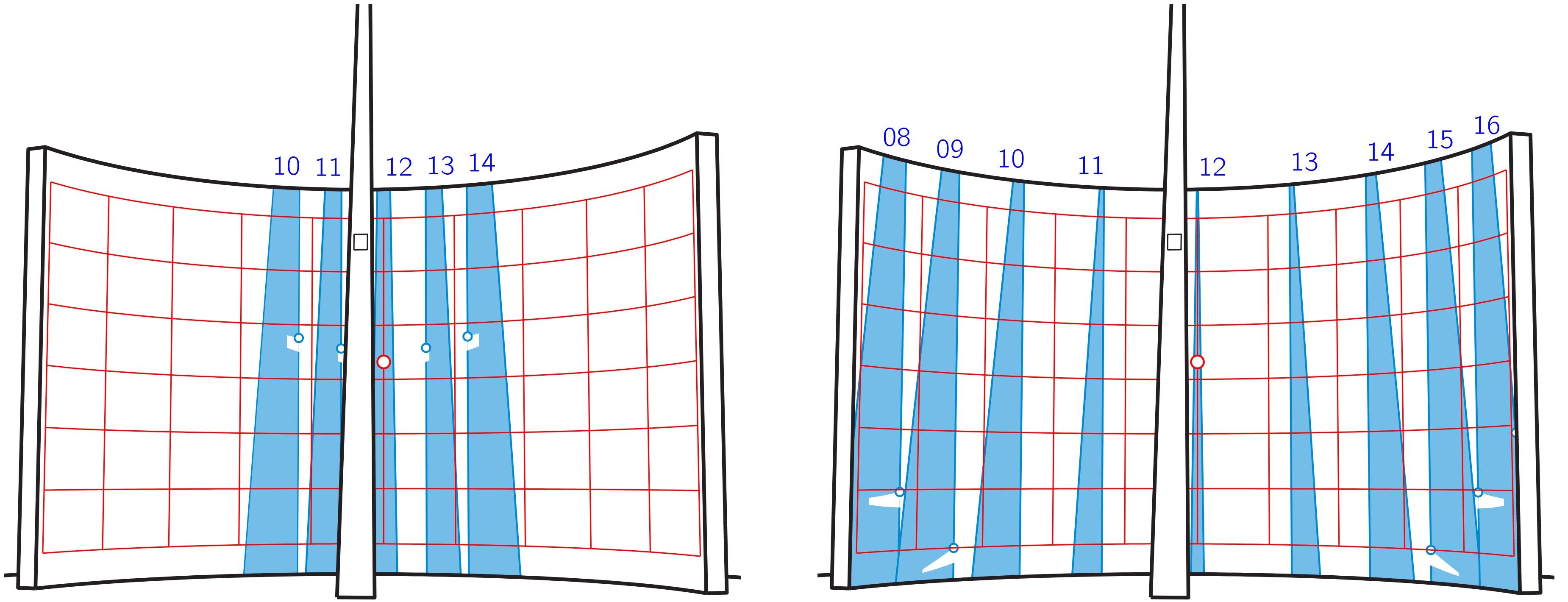
When I was contacted by the sponsoring organisation, there existed already the cylindrical wall (4 m high, opening exactly toward South).

There were also rough plans with the dimensions and placement of the ‘Gnomon’.

unit: 1 cm



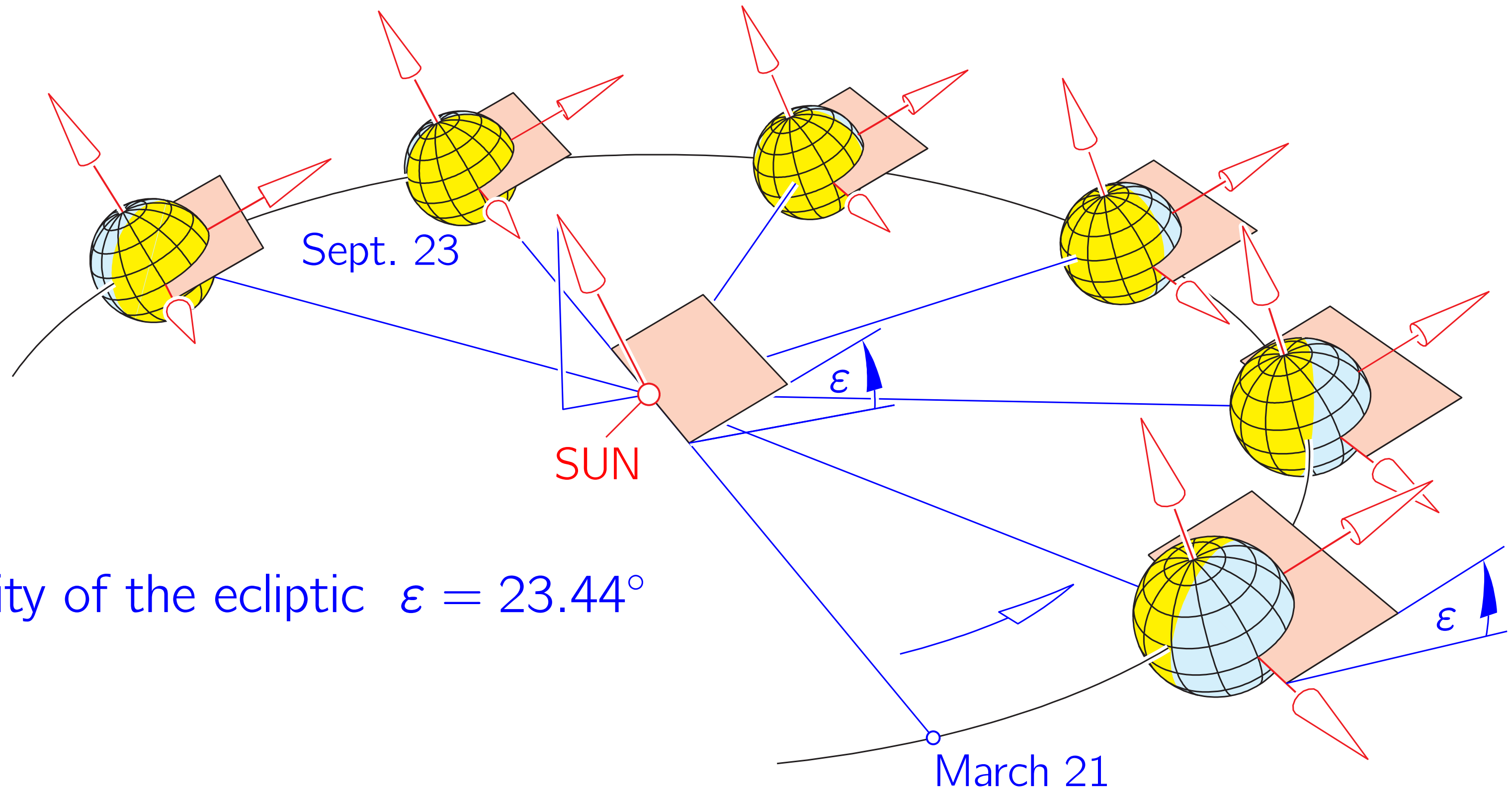
2. Why no sundial with shadows ?



Dec. 21 | June 6

These figures reveal that the shadows casted from the 'Gnomon' at the same daytime t differ over the year. The explanation follows below:

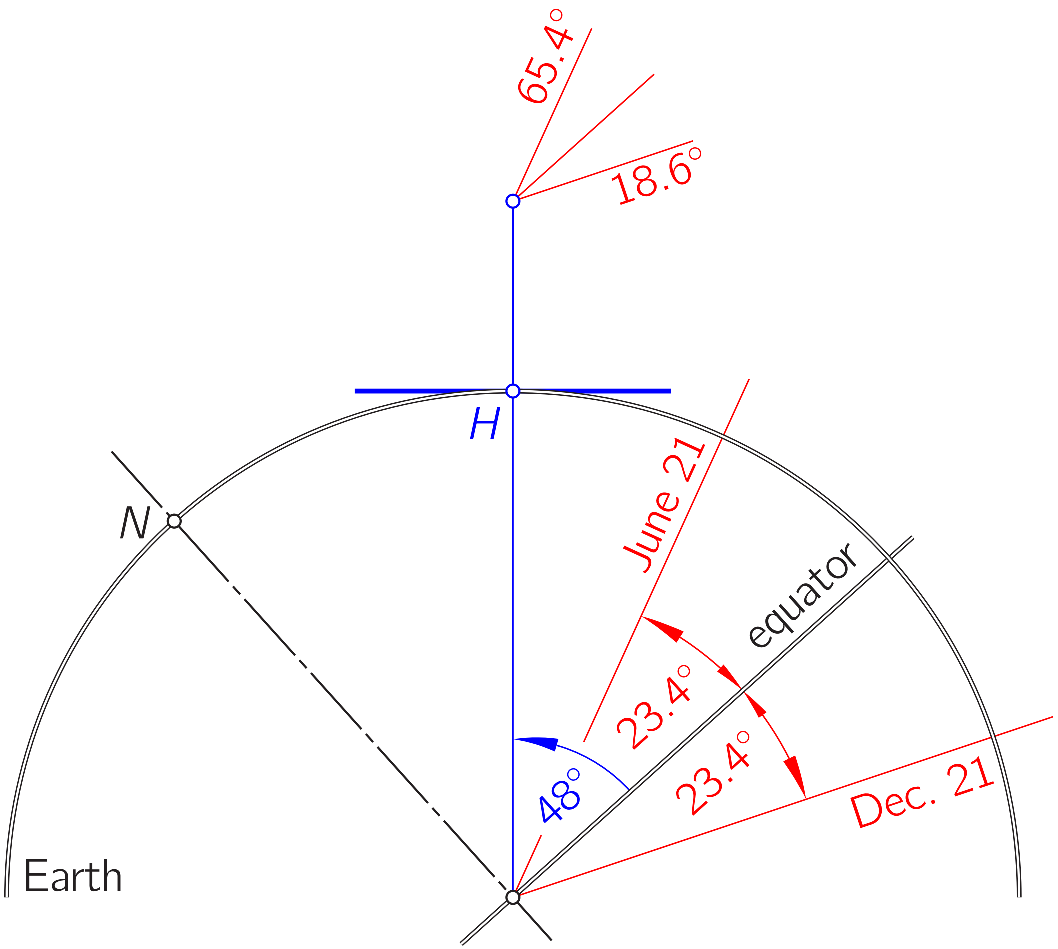
2. Why no sundial with shadows ?



obliquity of the ecliptic $\varepsilon = 23.44°$

Over the year the angle of elevation of the sun with respect to the equatorial plane varies between $-\varepsilon$ and ε, for $\varepsilon = 23.44°$ being the *obliquity of the ecliptic.*

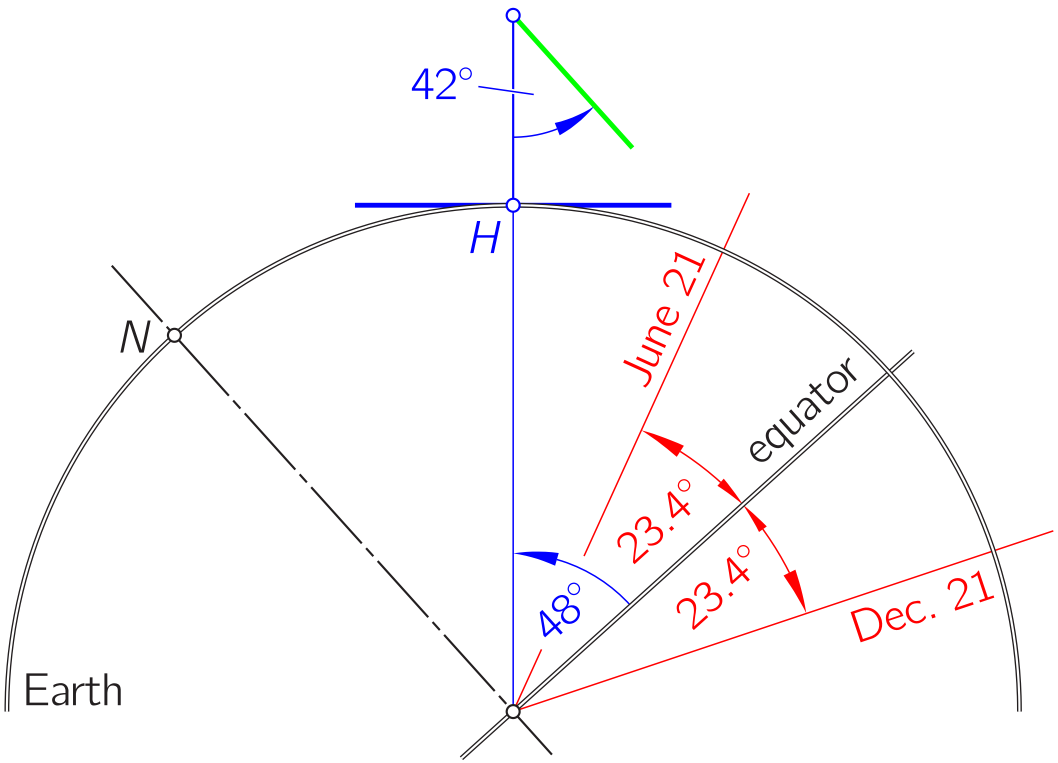
2. Why no sundial with shadows ?



This means for Heiligenkreuz (geogr. latitude 48.055°) that the maximum elevation per day ranges between 18.6° and 65.4°.

At the same daytime t ('true time') the sun is in the same meridian plane of the earth.

2. Why no sundial with shadows ?



The shadow of a stick at given local time t falls on the same hour-line independently from the season $\Longleftrightarrow$ the shadow plane (hour-plane) is parallel to the corresponding meridian plane.

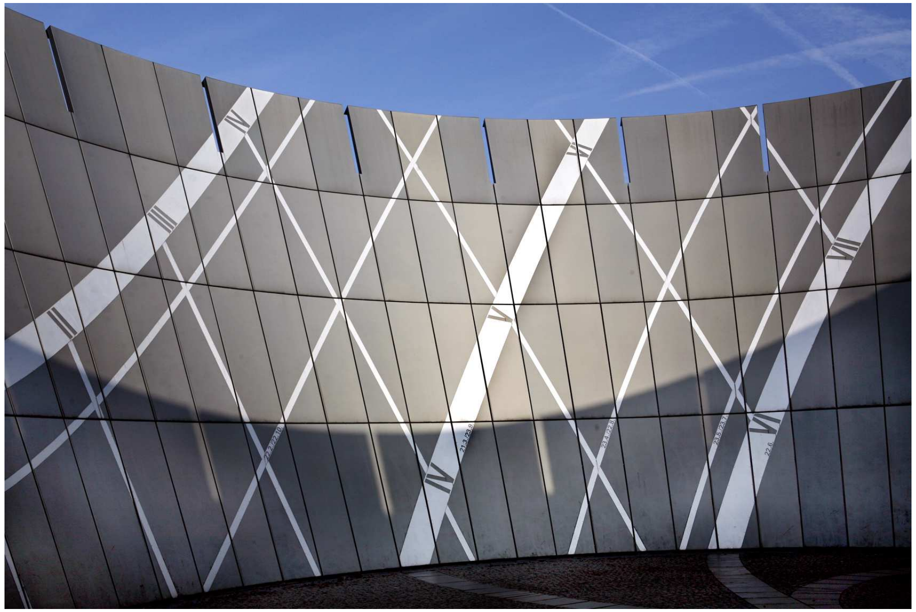
This holds for all $t \Longrightarrow$ the stick must be parallel to the earth's axis.

A line parallel to the earth's axis makes with a vertical wall (directed toward South) under 42°.

2. Why no sundial with shadows ?

Třeboň, Czech Republic

Traditional sundial with its gnomon parallel to the earth's axis

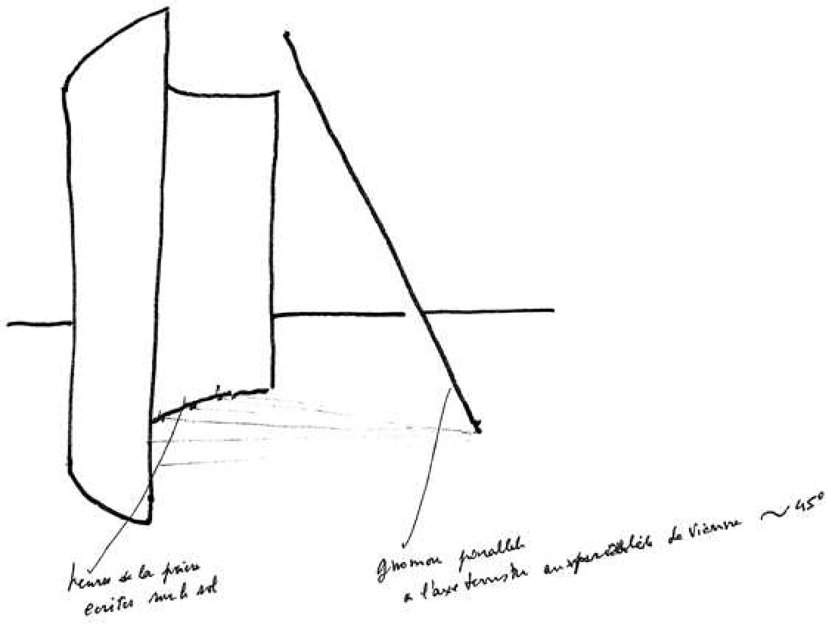


Sun-dial in Munich (subway station)

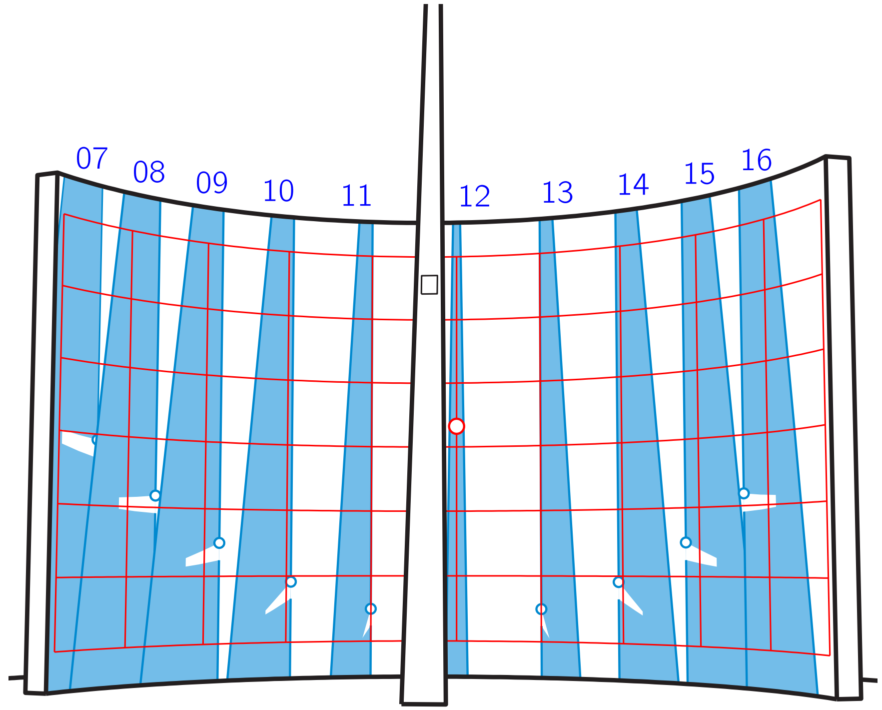
Based on hour- and date-lines the time as well as the date (two options) can be figured out.

2. Why no sundial with shadows ?

The shadow casted of the vertical 'Gnomon' at the same time t varies from day to day — except for $t = 12:00$.



This sketch reveals that the artist finally must have been sceptic about his original idea.



The 'Gnomon' with a hole shows the shadow of a single point. Therefore, by virtue of hour- and datelines the time could be extracted.

2. Why no sundial with shadows ?

But such a hole in the 'Gnomon', through which

on Jan. 6 at noon a sunbeam meets baby Jesus,

will **not** work as expected.

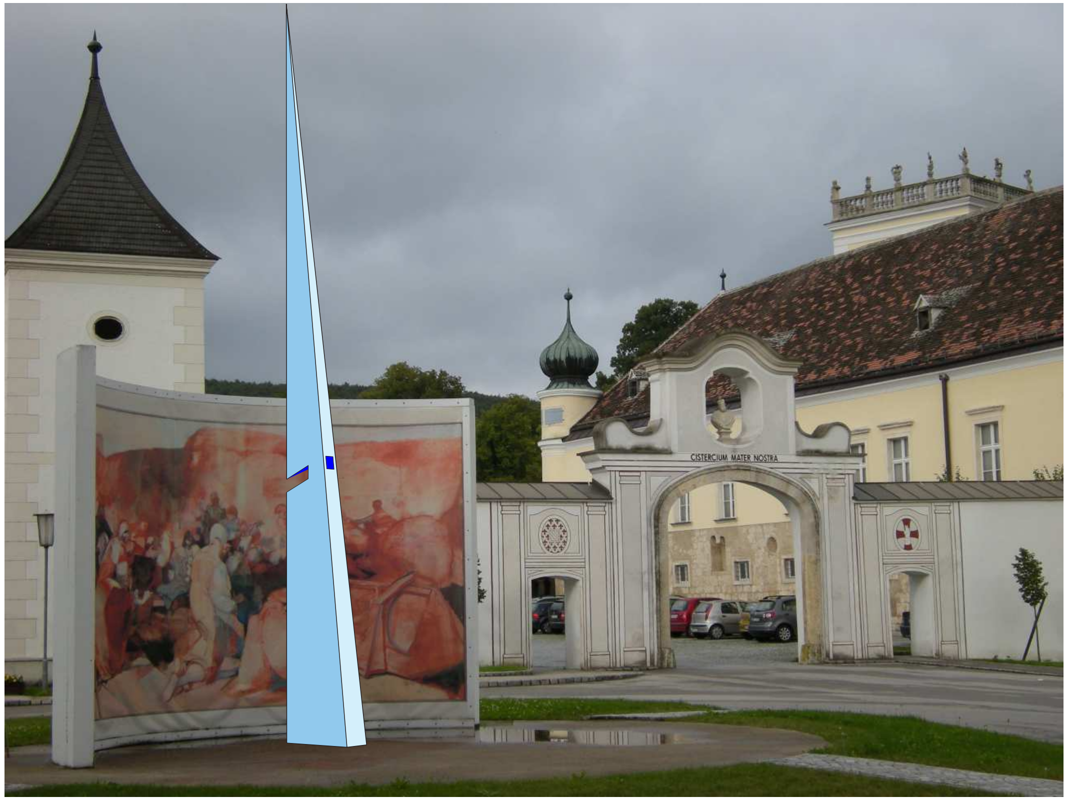
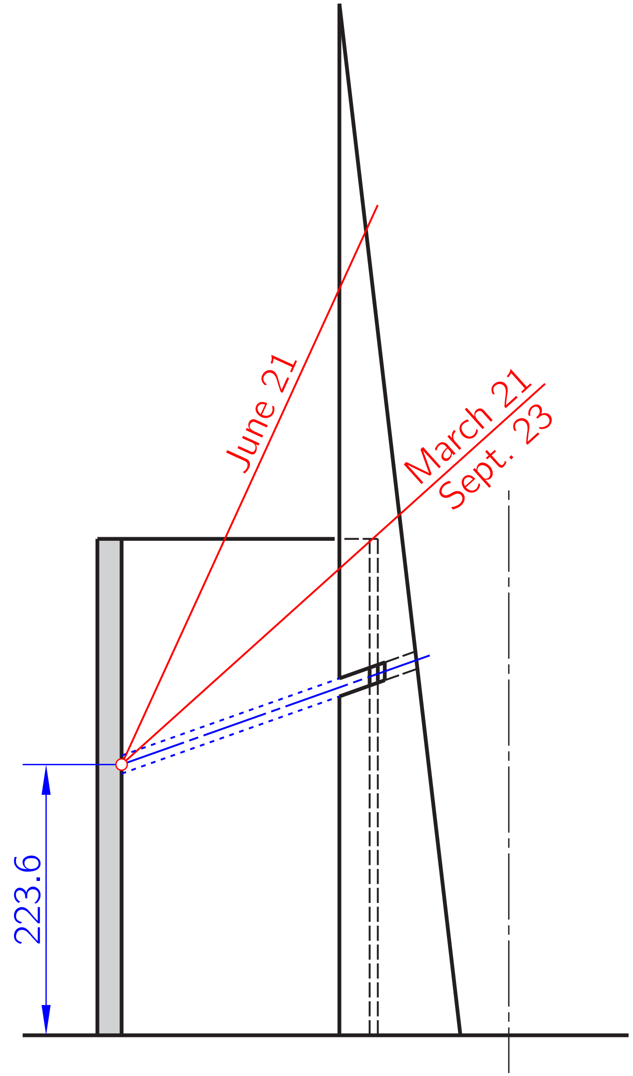


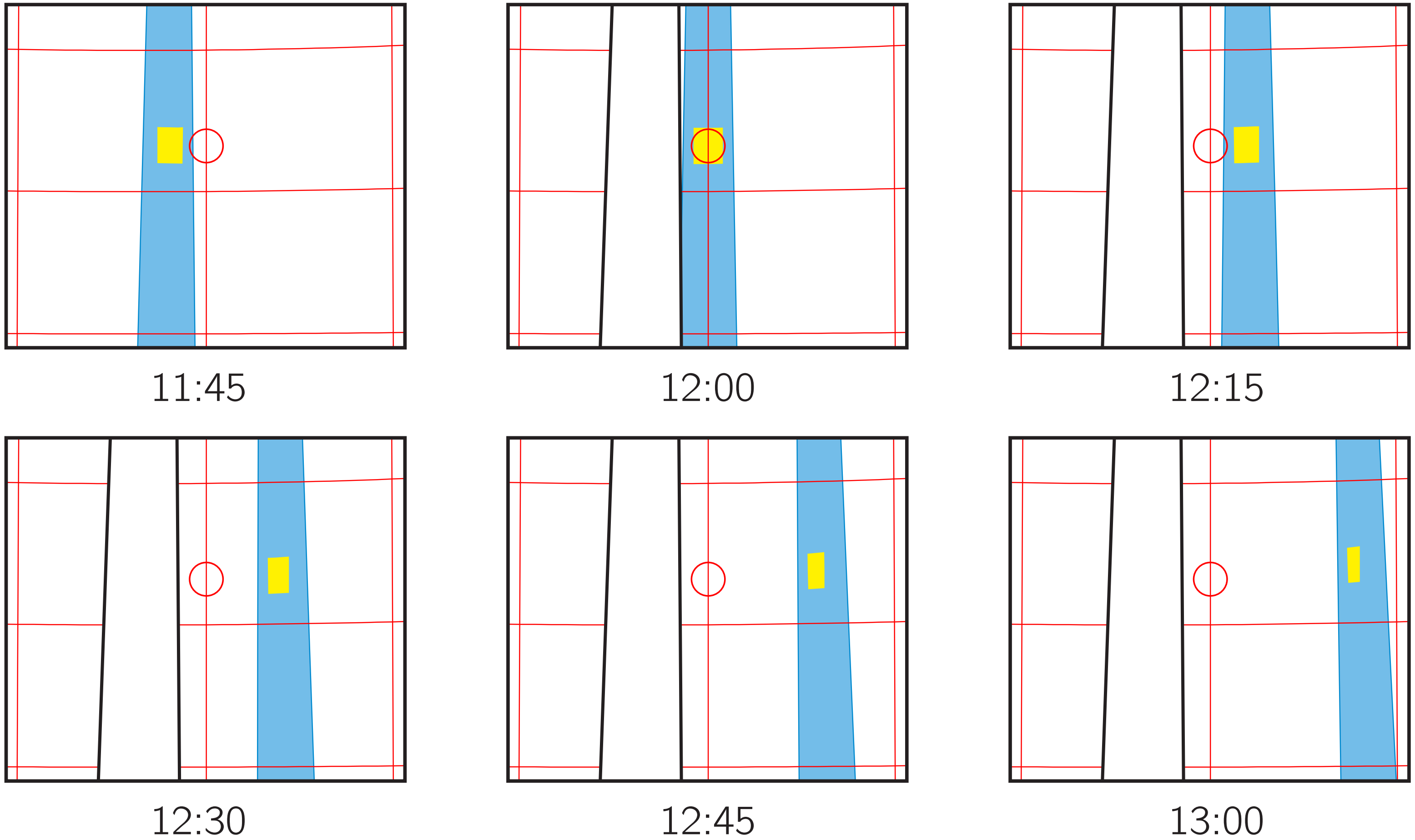
photo simulation: 'Gnomon' with hole

On a sunny day in winter the difference in brightness will not be sufficient. Furthermore, this sunbeam will also show up weeks before and afterwards.

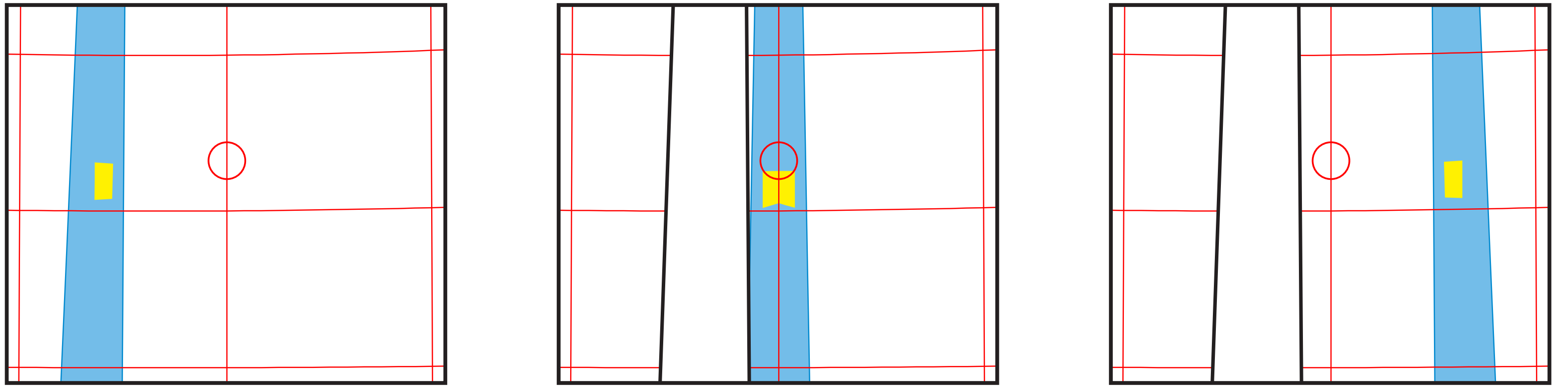
The size of the hole (width 12 cm, height 14 cm) is adjusted to the size of the depicted baby Jesus.



2. Why no sundial with shadows ?

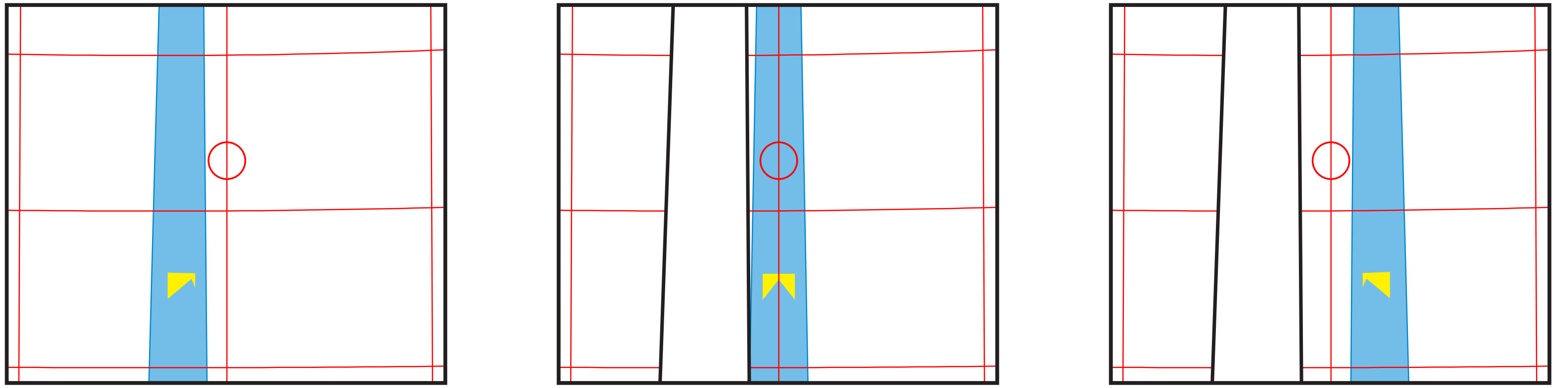


light effects on Jan. 6 and Dec. 6



11:15 12:00 12:45

two weeks later, on Jan. 20 as well as on Nov. 20



11:45 12:00 12:15

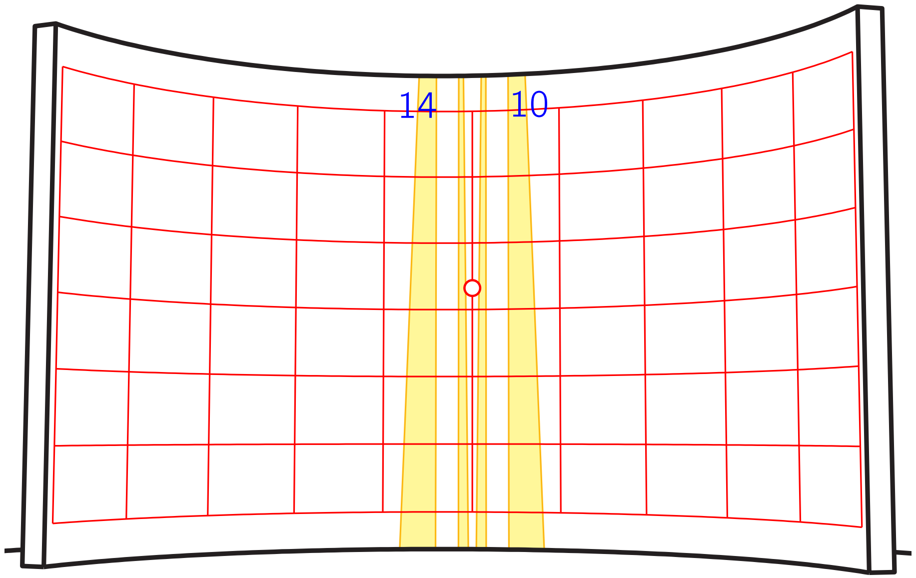
40 days later, on Feb. 15 as well as on Oct. 27

2. Why no sundial with shadows ?

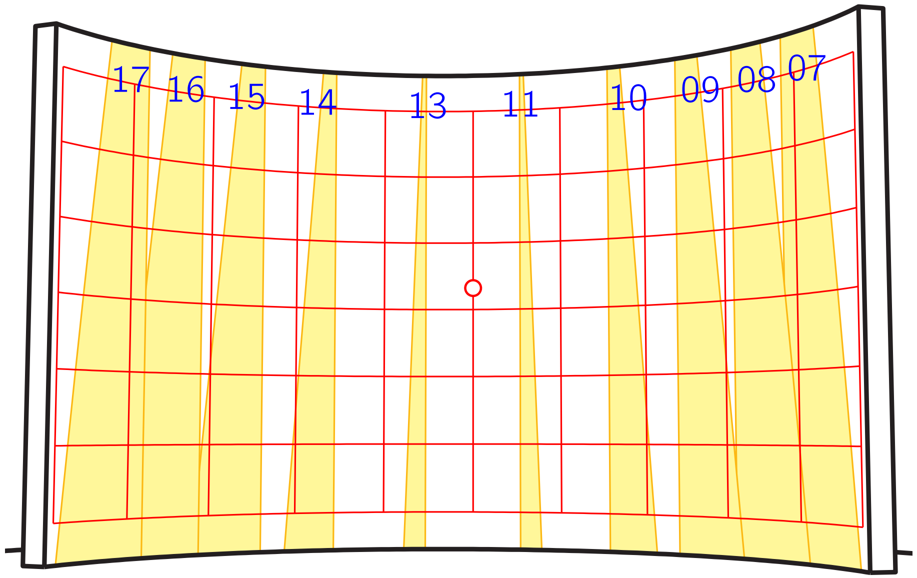
The illuminated spot caused by the hole in the 'Gnomon' will be visible around noon from end of October until midth of February.

Therefore an additional spotlight has been prepared for baby Jesus

3. Sundial based on the reflection of sunlight



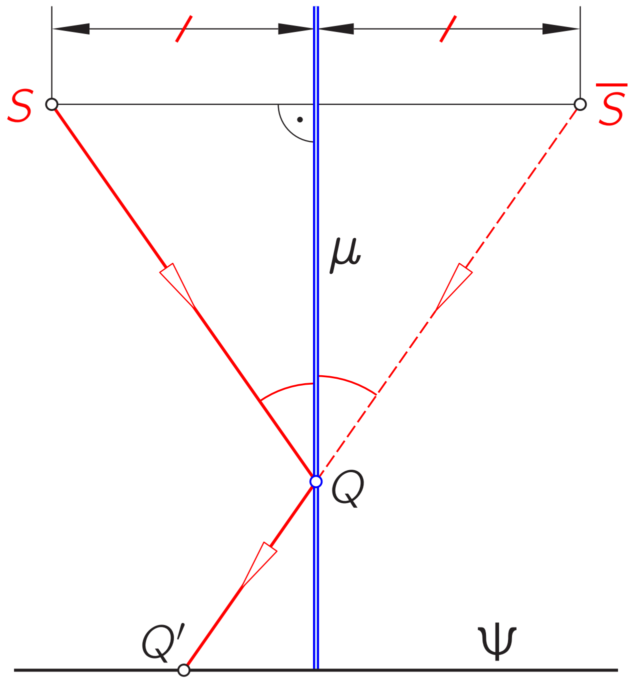
Dec. 21



June 6

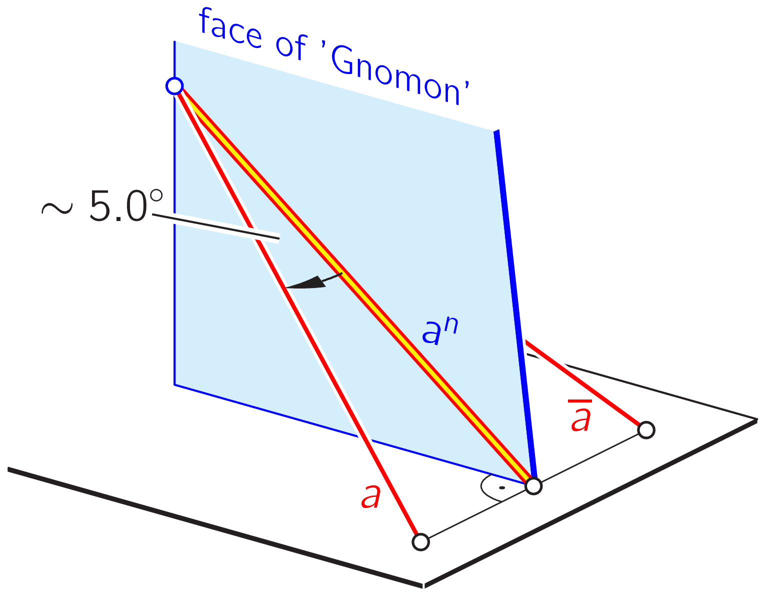
The reflection of the sunlight in the East- and Westface of the 'Gnomon' produces luminous stripes on the mosaic. Like the cast shadow the position at daytime t is not independent from the season.

3. Sundial based on the reflection of sunlight



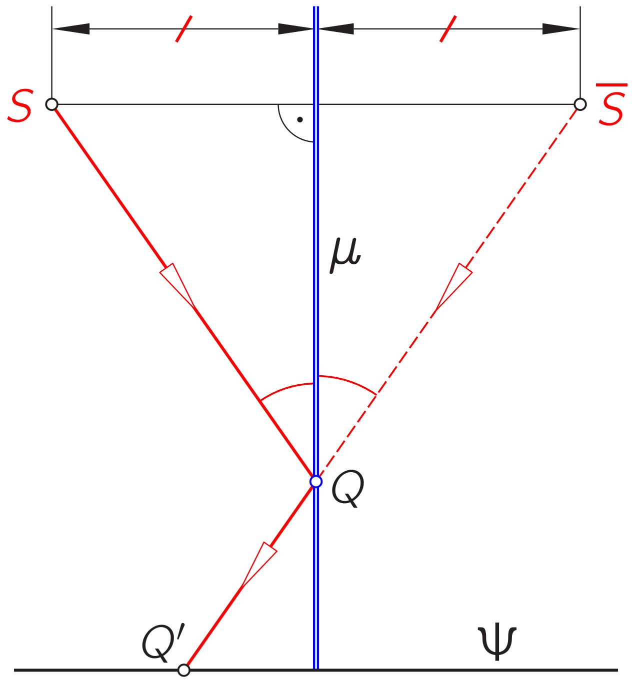
The luminous point Q' caused by reflection of a sunbeam in the plane μ equals the 'shadow' of Q w.r.t. the mirror $\overline{S}$ of the S.

The mirrors of the sunbeams meeting Q at the same time t all over the year lie in a plane through the mirror $\overline{a}$ of the line a which is parallel to the earth's axis.



However, $\overline{a}$ cannot serve as a reflecting stripe as it is not located in the 'Gnomon's face μ.

3. Sundial based on the reflection of sunlight



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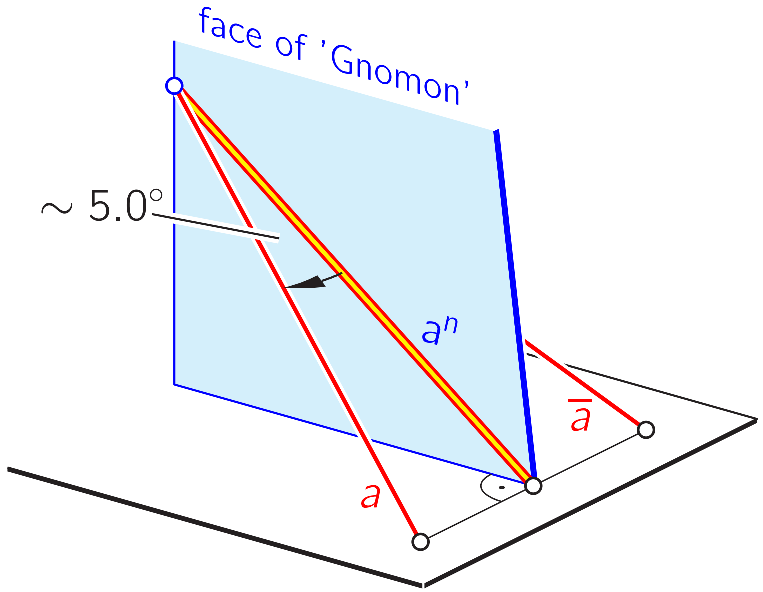
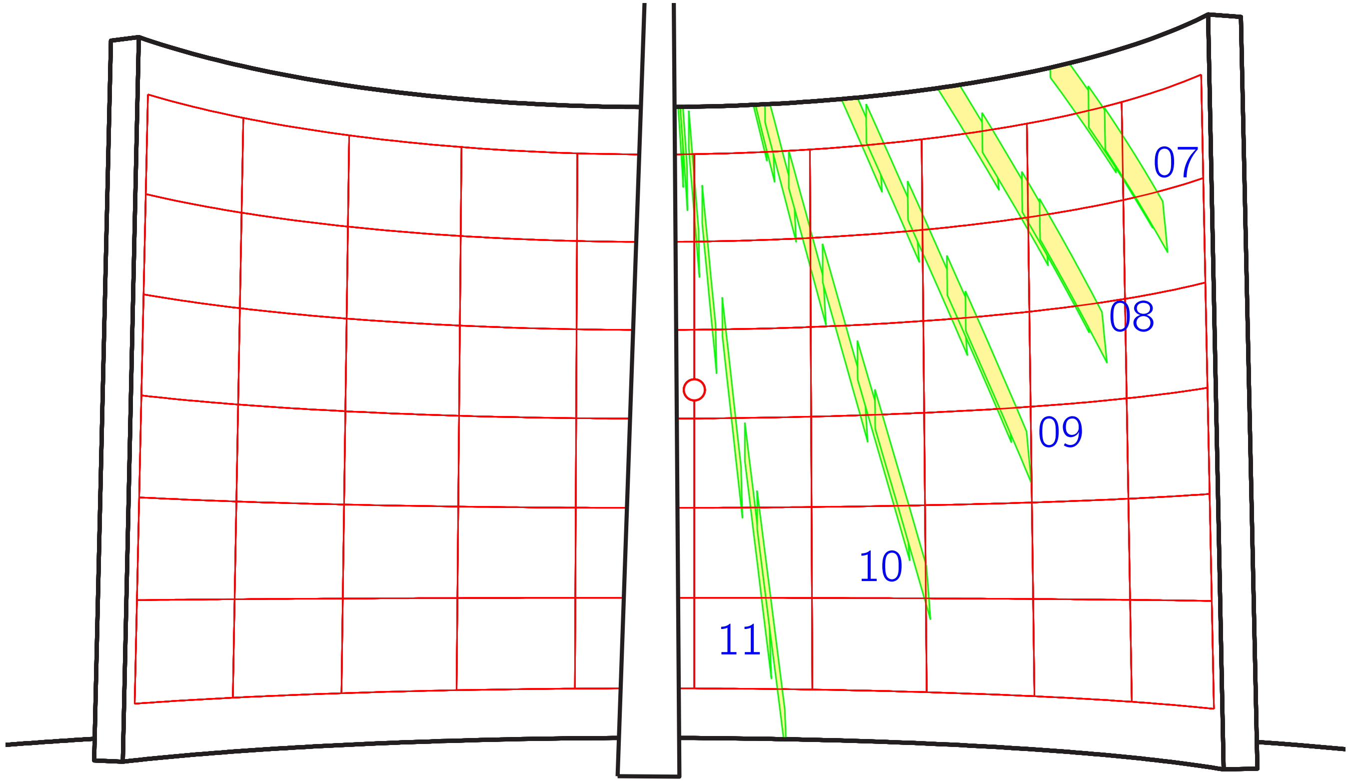


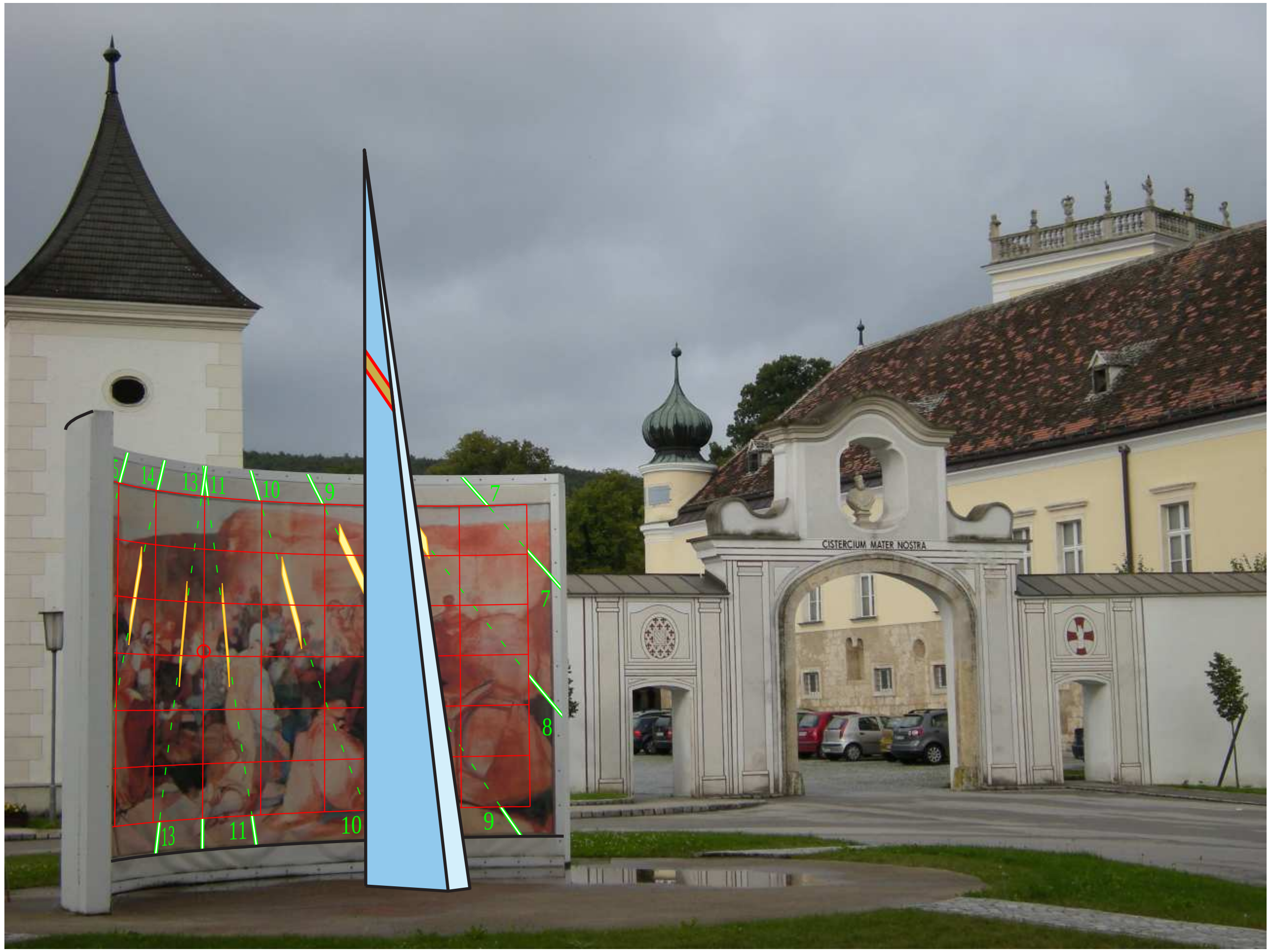
Best approximation: We replace $\overline{a}$ by its orthogonal projection a^n in μ which equals the projection of a.

3. Sundial based on the reflection of sunlight



This reveals: A reflecting stripe approximately parallel to the earth's axis generates on the cylindrical wall on the East- and Westface of the 'Gnomon' luminous stripes which for each t follow almost a 'hour-line' over the season (the earlier, the better).

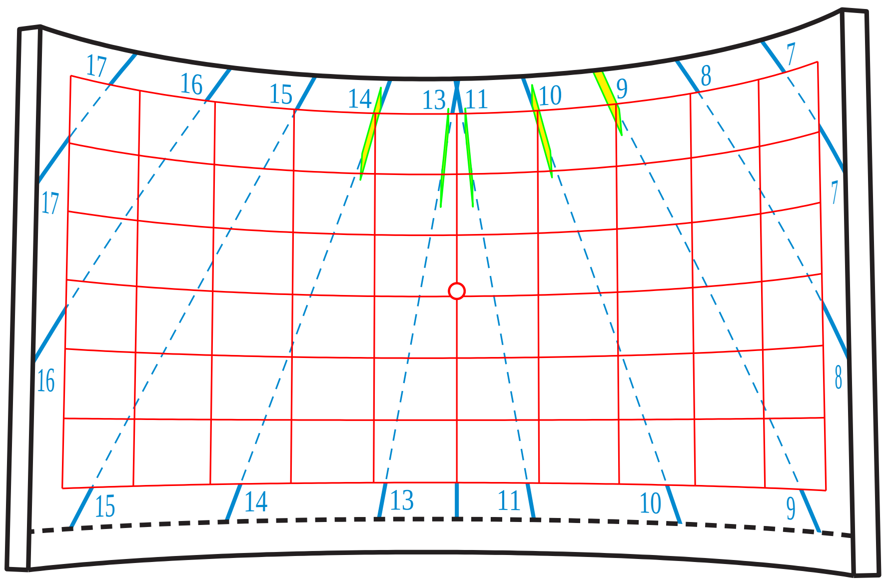
3. Sundial based on the reflection of sunlight



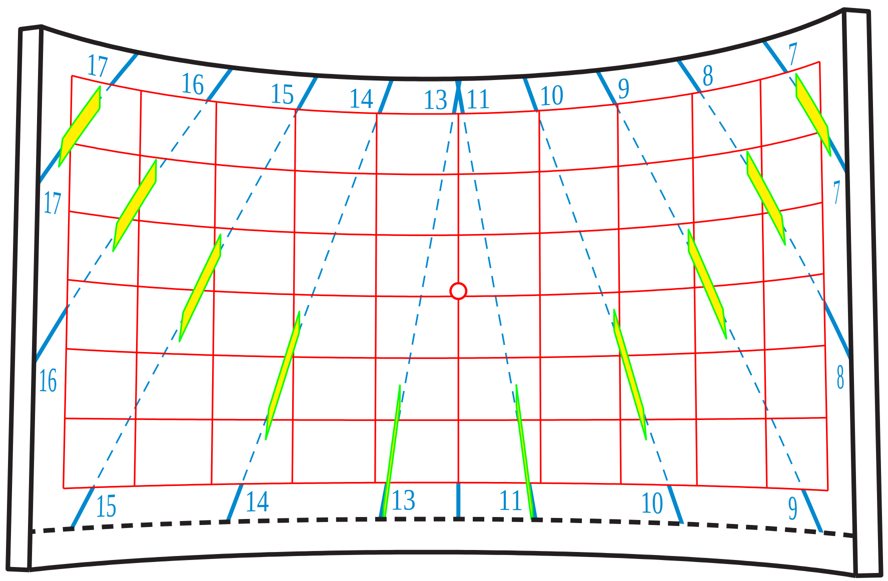
This photo simulation shows the 'Gnomon' with the reflecting stripe and the effects on September 9.

Remember: The reflections are like shadows of the stripe casted by the mirror $\overline{S}$ of the sun with respect to the face plane of the 'Gnomon'.

3. Sundial based on the reflection of sunlight



Feb. 21., Oct. 21



May 21, July 21

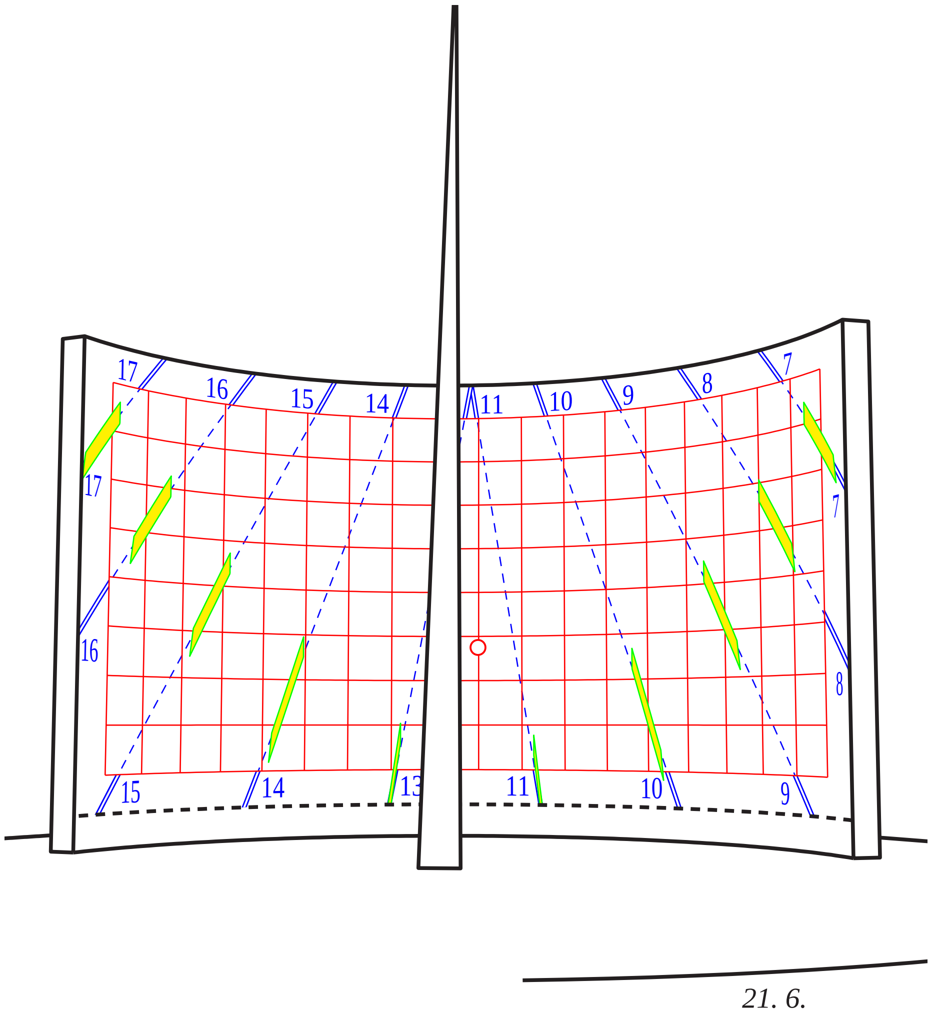
The exact position of the 'Gnomon' and the altitude of the reflecting stripe resulted from the request that even for summer- and winter solstices a luminous stripe should be visible on the curved wall.

3. Sundial based on the reflection of sunlight

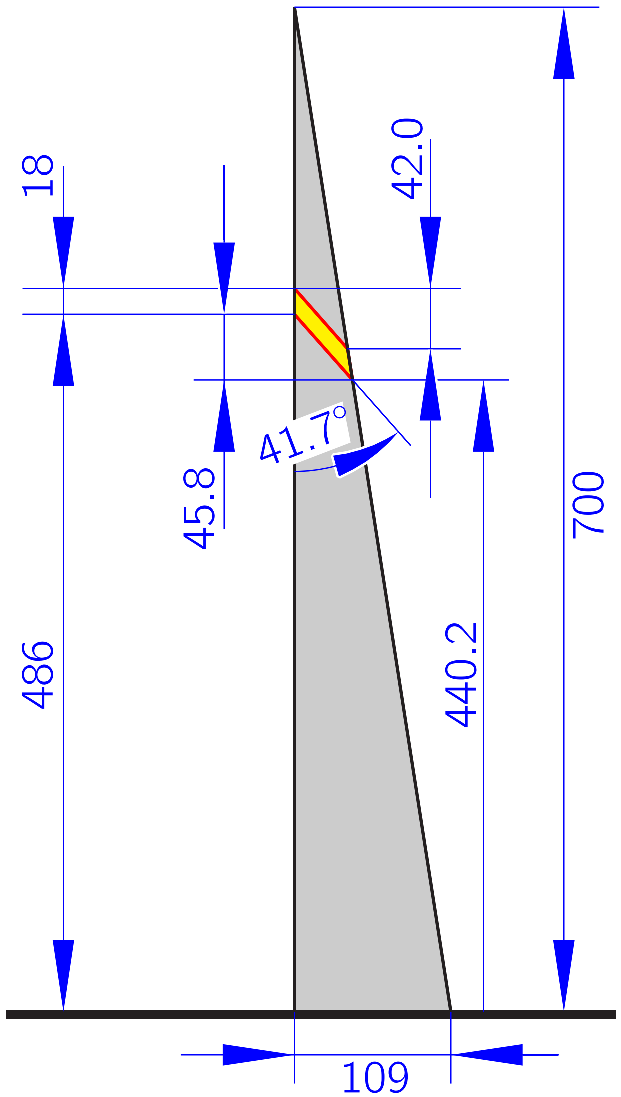
Due to the reason that

light (= "positive") instead of **shadow** (= "negative")

tells the time, was convincing for the head of the monastry, Abt Maximilian Heim, and he agreed – after shortening the 'Gnomon' from 8.5 m to 7.00 m.

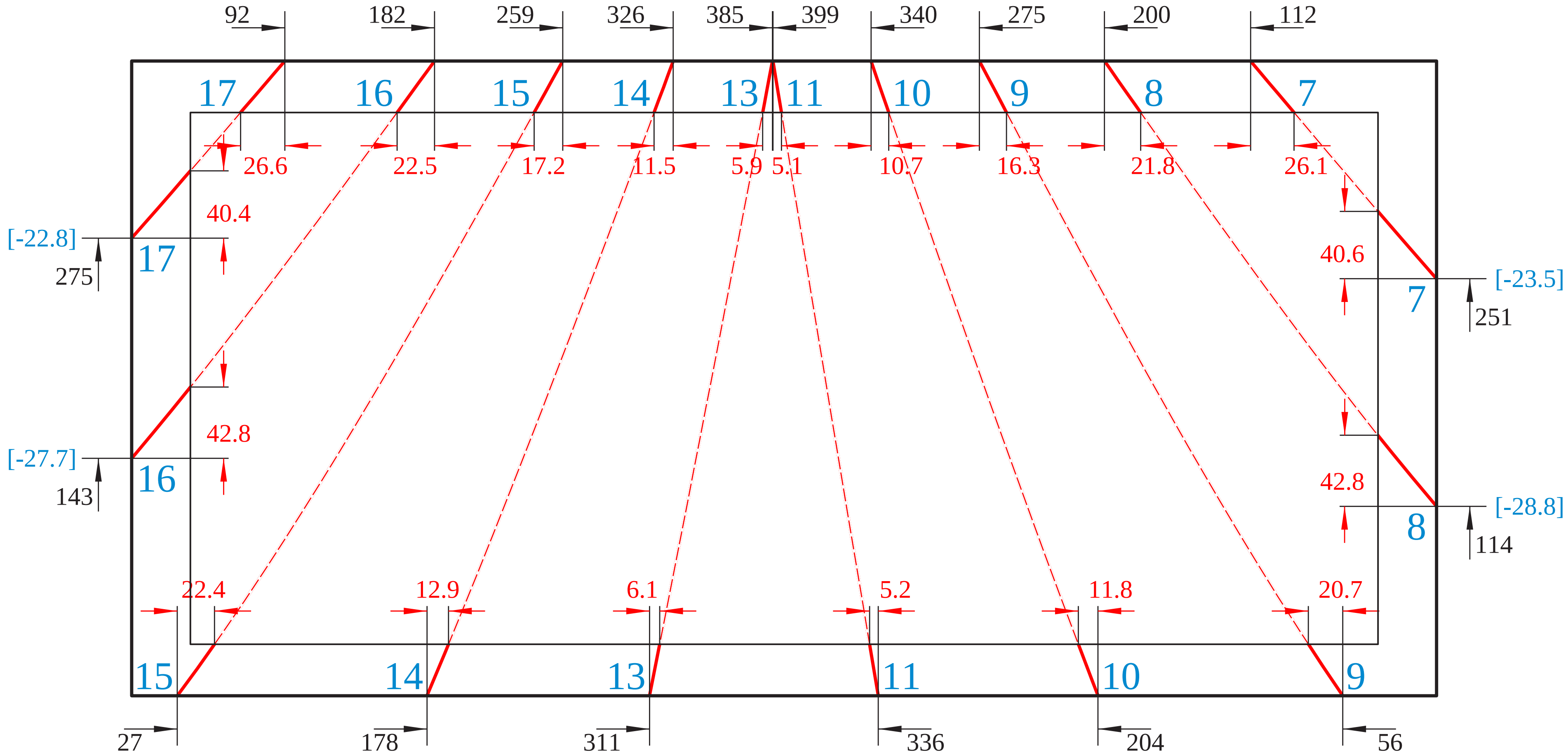


3. Sundial based on the reflection of sunlight



Original plan for workmen:
Front view of the 'Gnomon' with its reflecting stripes (unit 1 cm)

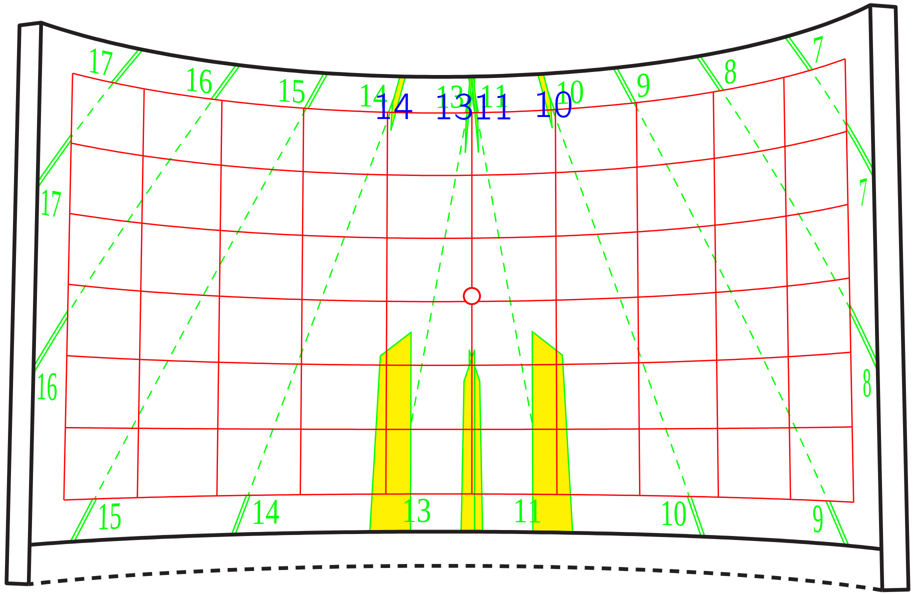
3. Sundial based on the reflection of sunlight



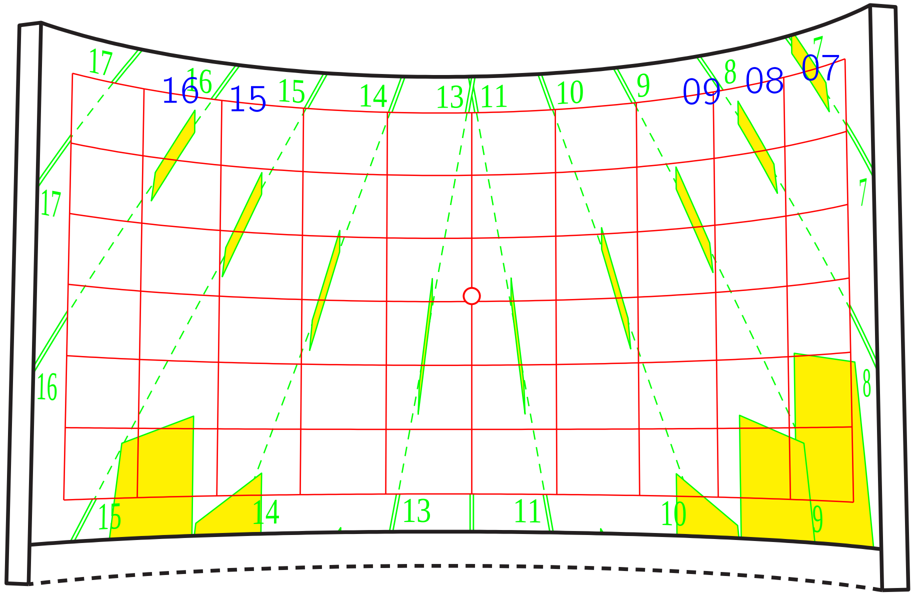
Plan for housepainter: Hour-lines under consideration of the geographic longitude 16.05°

August 15. Unfortunately, the lower portions of the faces are slighly bent due to production errors. This causes irregular light reflexes which can confuse visitors.

3. Sundial based on the reflection of sunlight



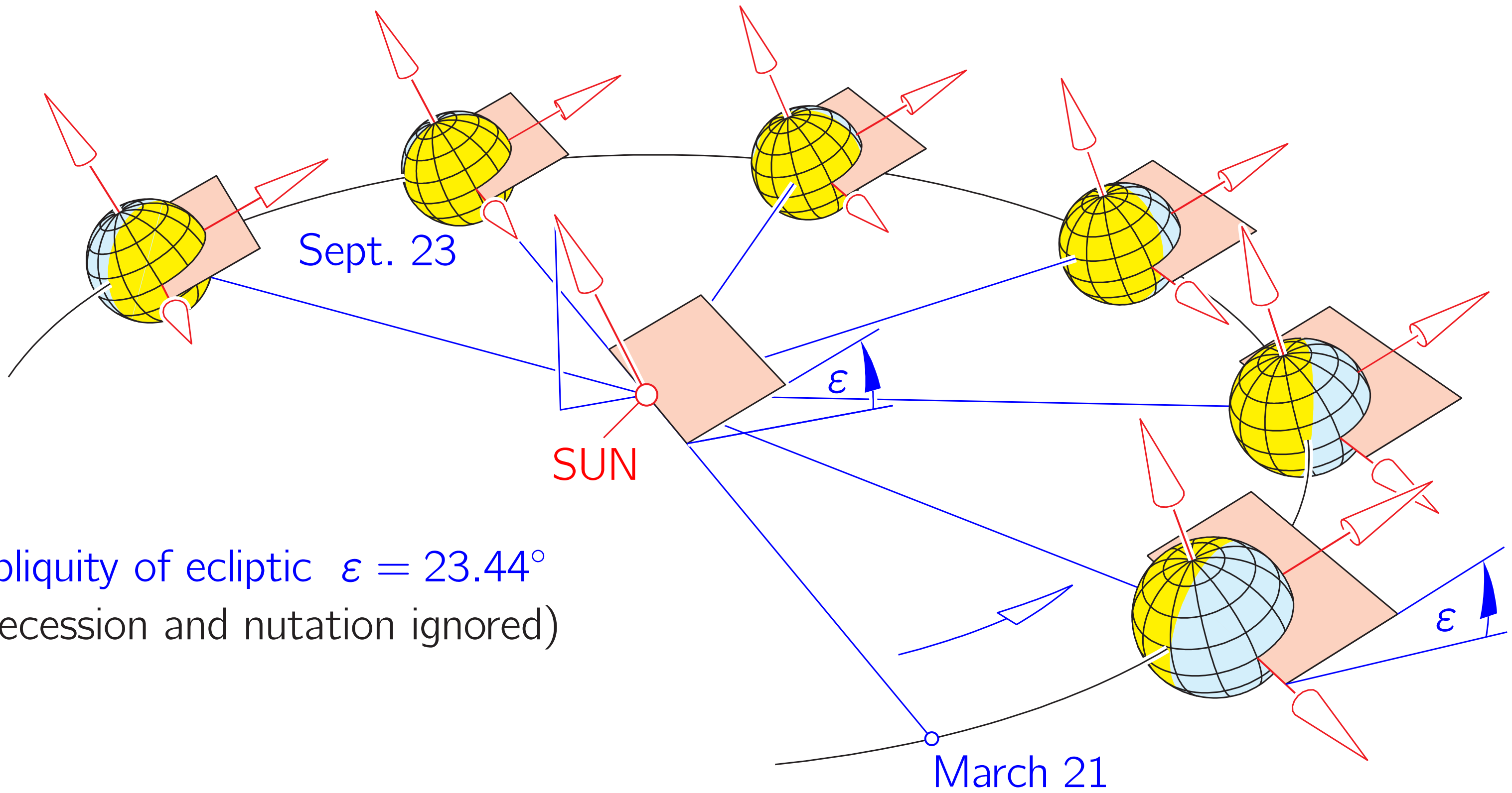
Jan. 21, Nov. 21



Apr. 21, Aug. 21

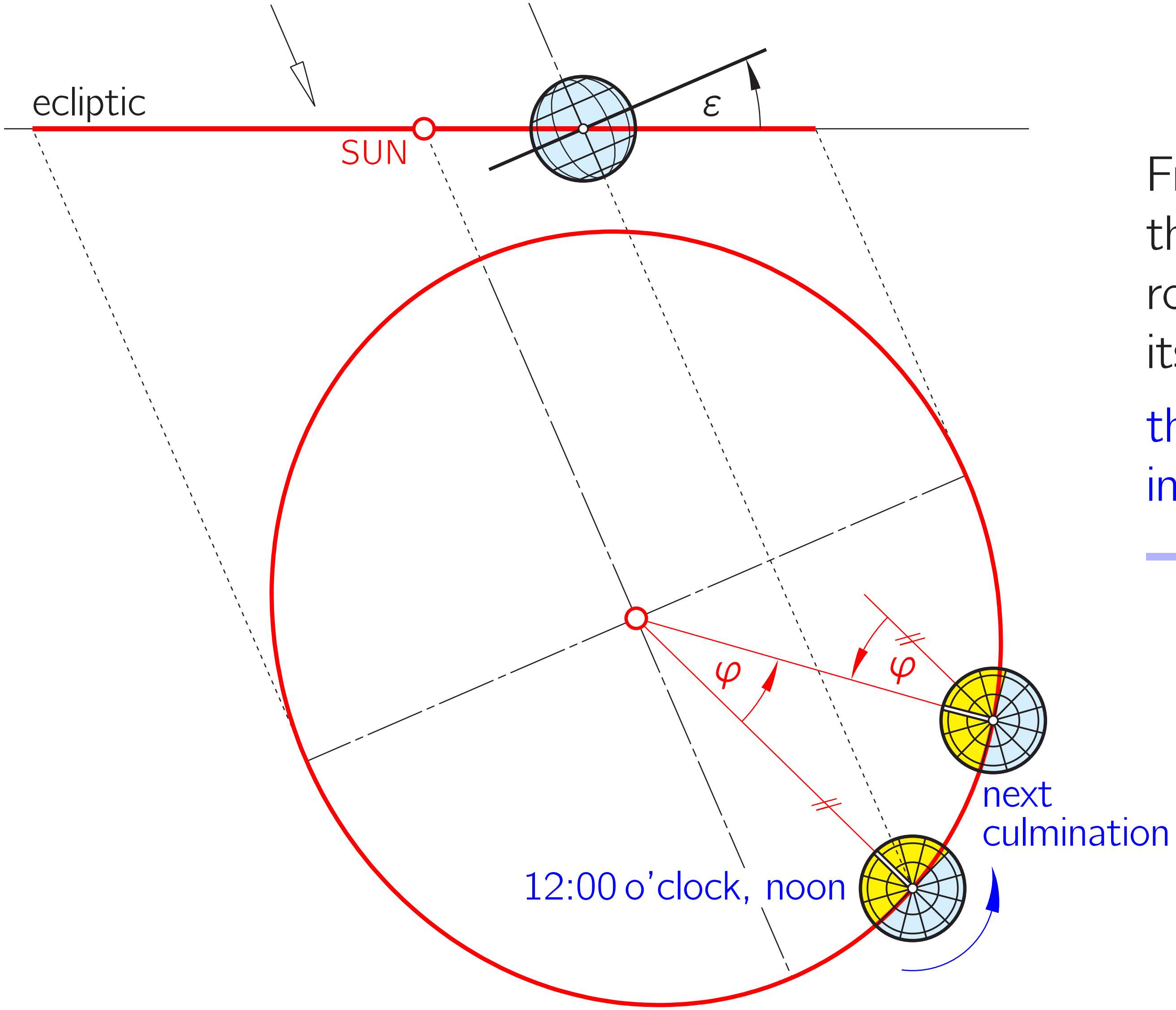
Lowering the unpolished part of the flanks by 50 cm would slighly improve the situation

4. On the precision of sundials



Obliquity of ecliptic $\varepsilon = 23.44°$
(precession and nutation ignored)

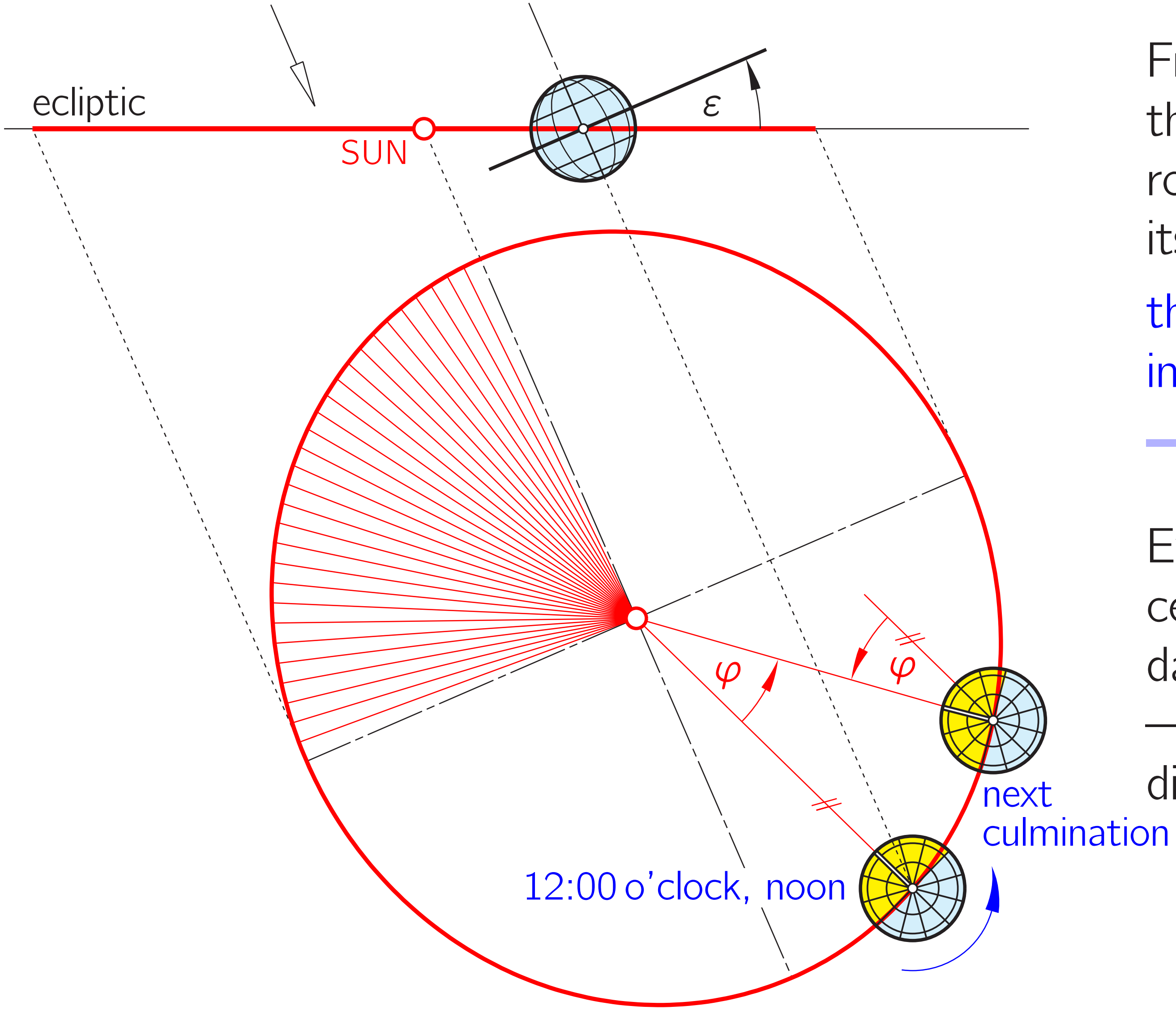
We project the trajectory of the earth orthogonally into the equator plane in order to inspect the rotation of the earth about its axis in true shape.



From one sun culmination to the next one the earth must rotate about $360° + \varphi$ about its axis $\Longrightarrow$

the days (in true time) differ in length !

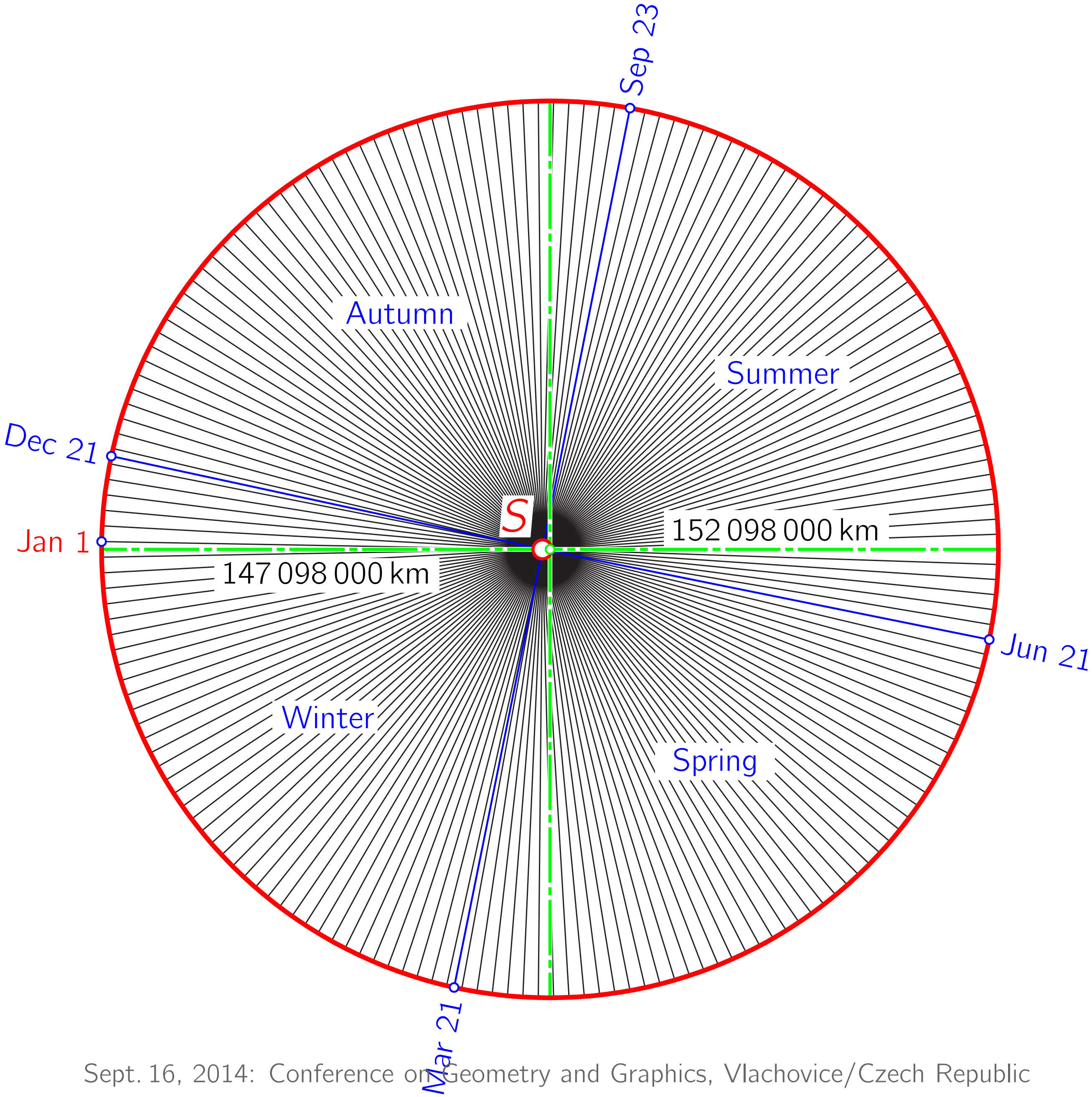
(image not scaled)



From one sun culmination to the next one the earth must rotate about $360° + \varphi$ about its axis $\Longrightarrow$

the days (in true time) differ in length !

Even for a circular path the center angles φ swept per day vary from day to day — because of the affine distorsion.



Kepler's First and Second Law: The path of the earth around the sun is an ellipse, traced with **constant areal velocity**.

(2 days time-interval for sub-division).

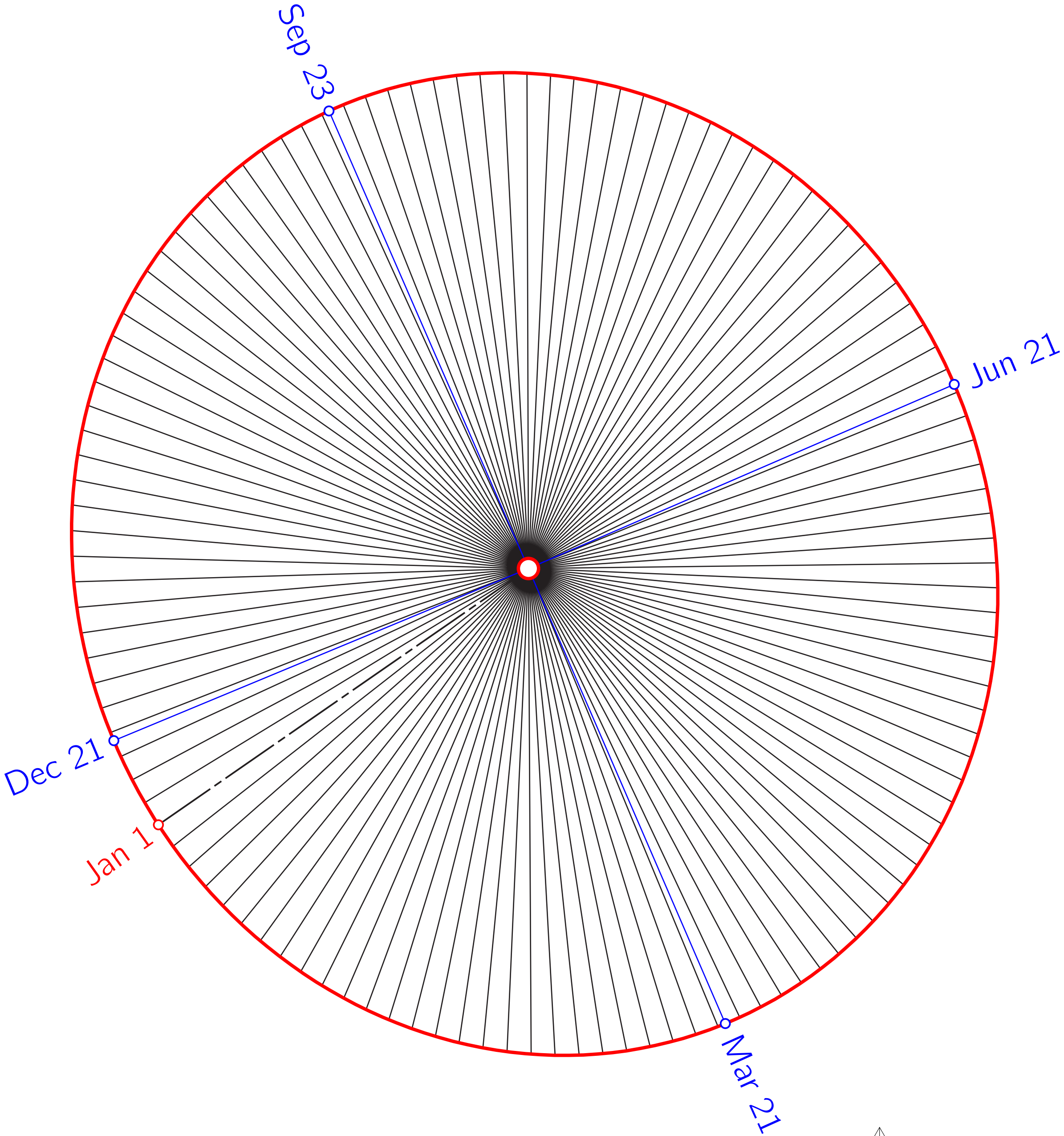
data:

$$a = 149.598\,10^6\,\text{km}$$
$$b = 149.577\,10^6\,\text{km}$$
$$e = 2.500\,10^6\,\text{km}$$
$$\varepsilon = e/a = 0.01671$$

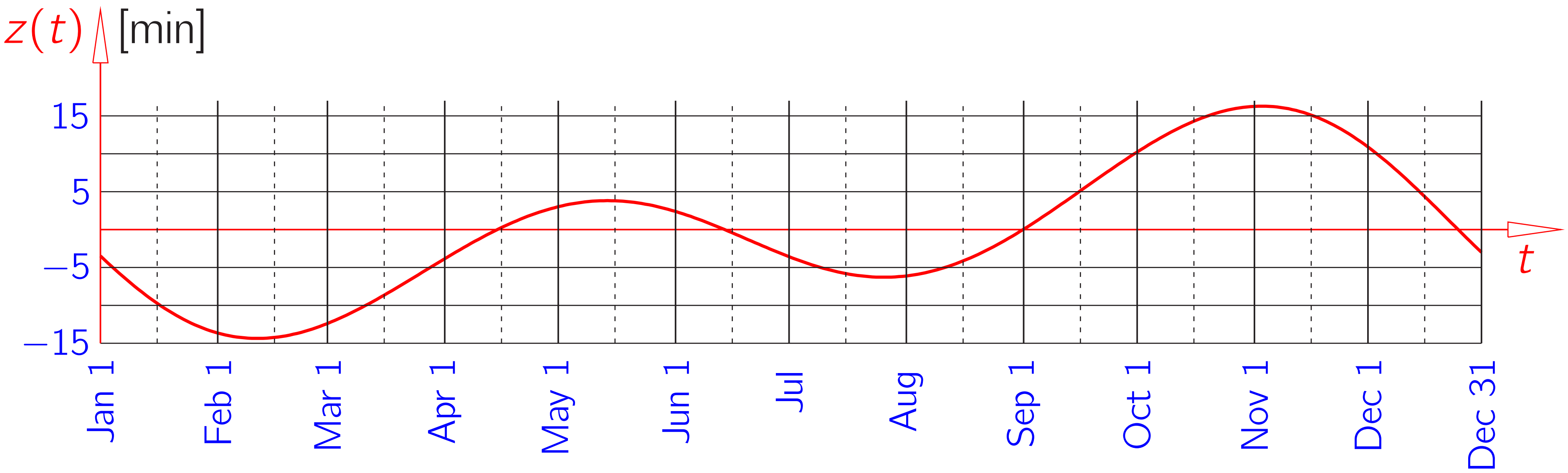
Auxiliary view of the true ecliptic, which again is traced with constant areal velocity.

3 days time-intervall (mean time) for subdivision

The deviation of the center angle φ from the mean value $360/365^\circ$ causes the **Equation of Time**.

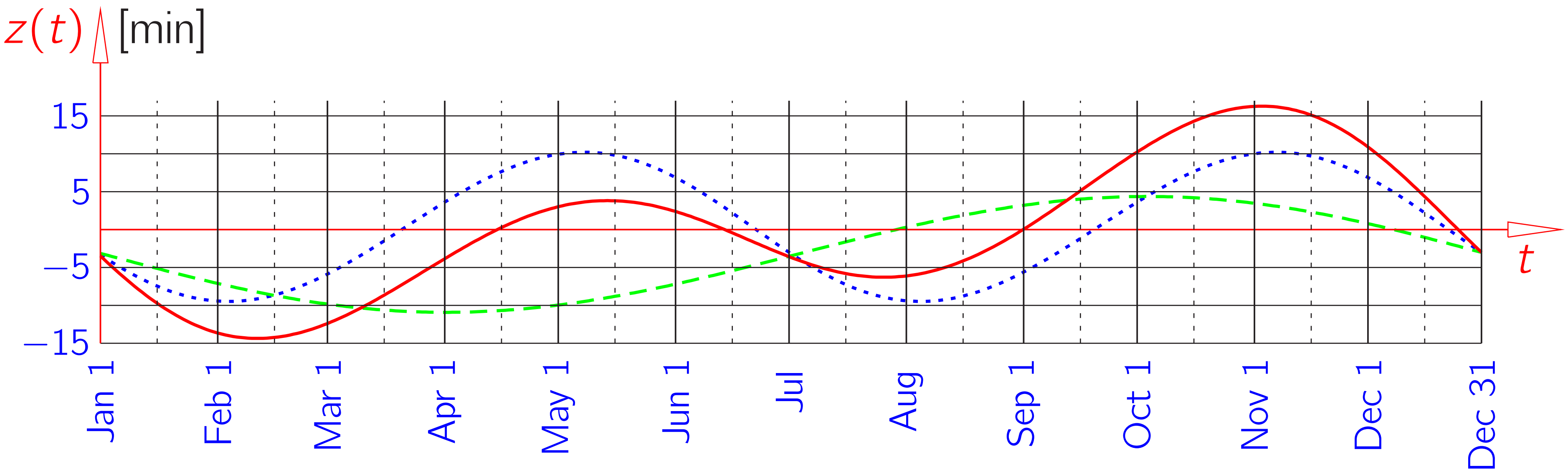


4. On the precision of sundials



Equation of Time: true time (sun) = mean time (clock) + $z(t)$

4. On the precision of sundials



Equation of Time: true time (sun) = mean time (clock) + $z(t)$

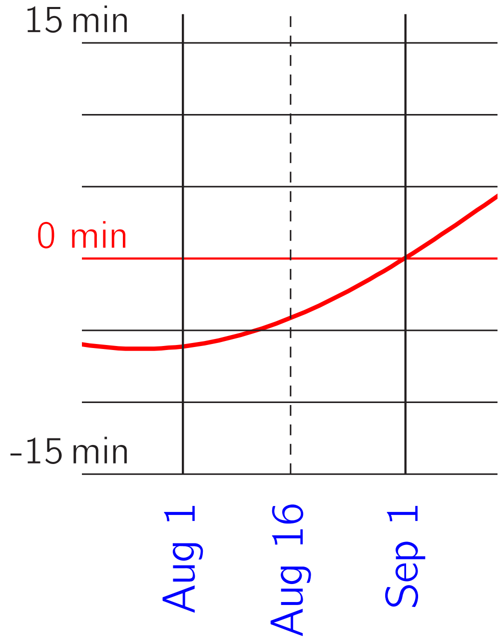
· · · · · · · · · Equation of Time for circular path ($e = 0$)

- - - - - - - Equation of Time without Obliquity of ecliptic ($\varepsilon = 0$)

City tower 'Alter Michel' in Munich:
The sun-dial pays already attention to the Equation of Time. Instead of a hour-line the tip of the shadow defines the time.

4. On the precision of sundials

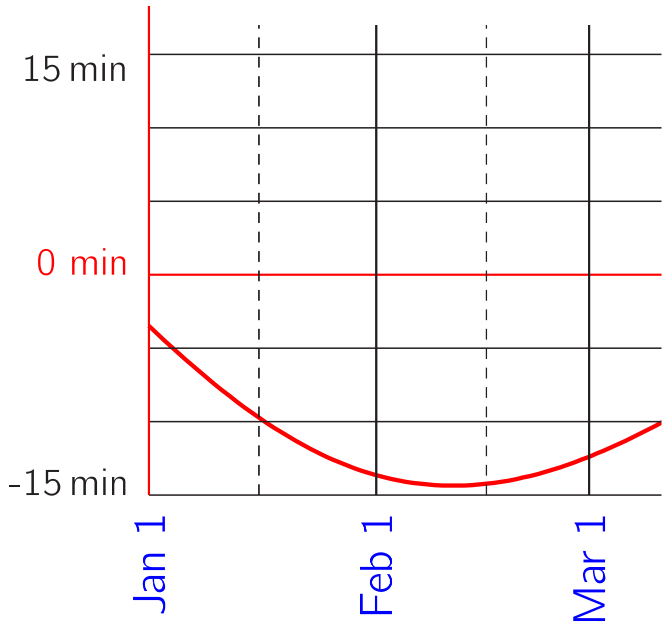
Equation of Time:



Example 1, Aug. 15: The reflection indicates $\sim$ **8:05 a.m.** ; European Summertime; the Equation of Time gives $z = -5\,\text{min}$ (true time − mean time) $\Longrightarrow$ result: $\sim$ **9:10 a.m.**

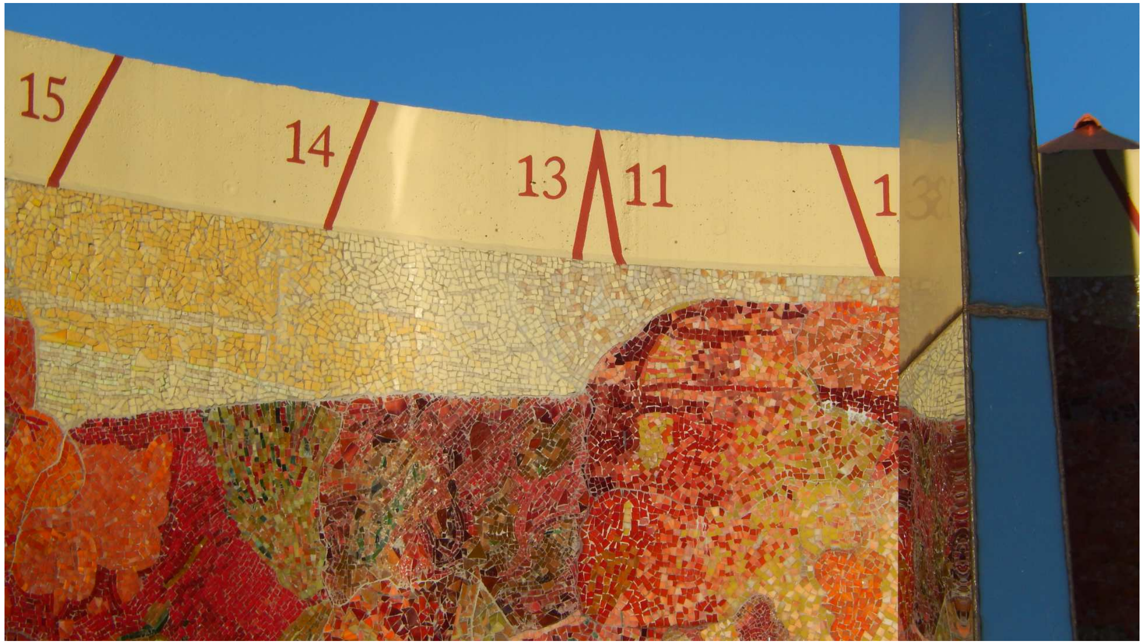
4. On the precision of sundials

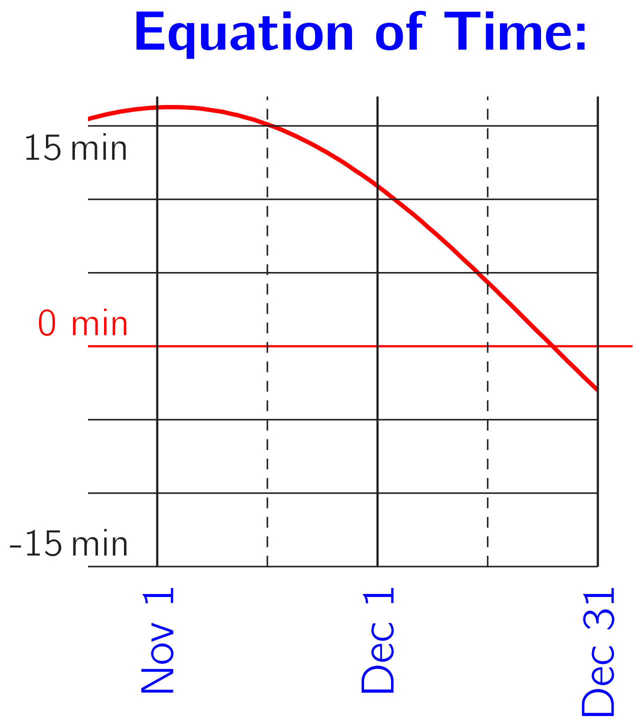
Equation of Time:



Example 2, Feb. 13: The reflection indicates $\sim$ **2:46 ? p.m.** ; European time; the Equation of Time gives $z = 15\,\text{min}$ (true time − mean time) $\Longrightarrow$ measured time: $\sim$ **3:01 p.m.**

4. On the precision of sundials

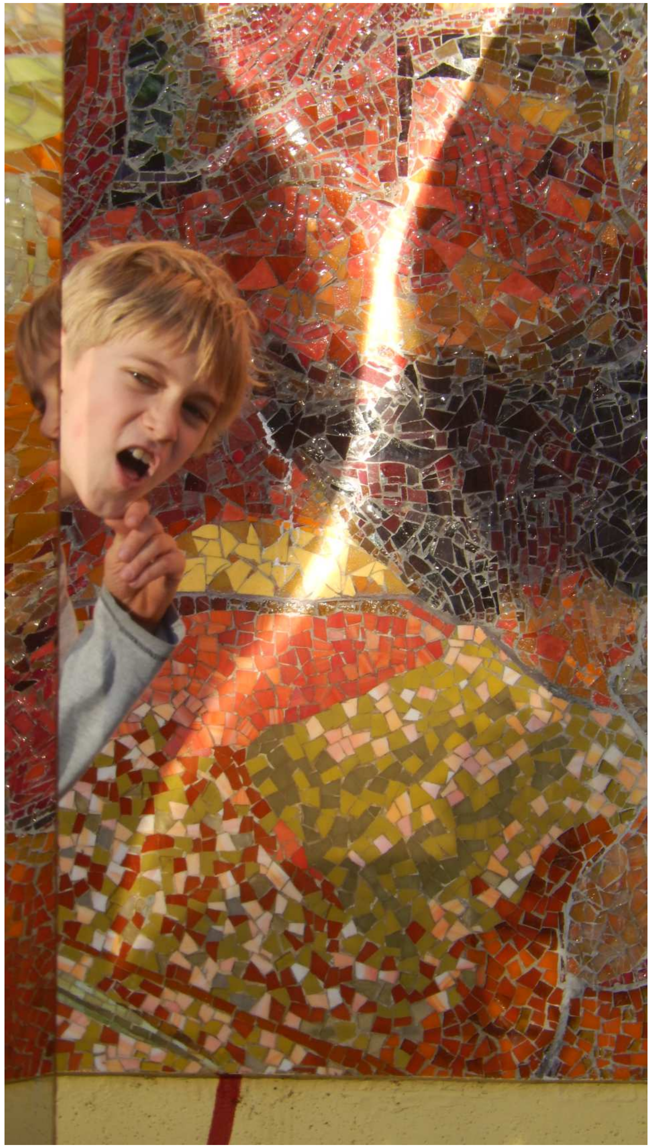




Example 3, Dec. 25: The reflection indicates $\sim$ **1:51 ? p.m.** ; European time; the Equation of Time gives $z = -1\,\mathrm{min}$ (true time − mean time) $\Longrightarrow$ measured time: $\sim$ **1:50 p.m.**

Consegration September 9, 2012

Abt Maximilian, bishop Laun (Salzburg), Abt Gregor, and "sun-reflection-dial"



Thank you for your attention!

Literatur

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