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DESCRIPTIVE GEOMETRY IN TODAY'S ENGINEERING CURRICULUM

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Summary

This is a pleading for Descriptive Geometry, a subject of basic importance for any engineering education. From the very first, Descriptive Geometry has been a method to study 3D geometry through 2D images thus offering insight into structure and metrical properties of spatial objects, processes and principles. The education in Descriptive Geometry provides a training of the students' intellectual capability of space perception. Drawings are the guide to geometry but not the main aim.

Key words: *Descriptive Geometry, space perception*

NACRTNA GEOMETRIJA NA SUVREMENOM STUDIJU TEHNIKE

Sažetak

Ovo je rasprava o Nacrtnoj geometriji, temeljnom predmetu svakog inženjerskog obrazovanja. Oduvjek je Nacrtna geometrija bila metoda kojom se trodimenzionalni prostor proučava pomoću dvodimenzionalnih slika, čime se daje jasna predodžba strukturnih i metričkih svojstava prostornih objekata i prostornih postupaka. Obrazovanje u Nacrtnoj geometriji omogućuje razvijanje sposobnosti prostornog predočavanja. Crtež je sredstvo u geometriji, ali ne i glavni cilj.

Ključne riječi: *Nacrtna geometrija, prostorno predočavanje*

1. Introduction

The aim of my presentation is to explain what Descriptive Geometry is good for, a subject, which in the hierarchy of sciences is placed somewhere within or next to the field of Mathematics, but also near to Architecture, Mechanical Engineering, and Engineering Graphics. I start with definitions and continue with a few examples in order to highlight that Descriptive Geometry provides a training of the students' intellectual capability of space perception (note the diagram in Fig. 9) and is therefore of incotestable importance for all engineers, physicians and natural scientists.

2. How to define 'Descriptive Geometry'

In many American textbooks on Engineering Graphics, e.g. [2, part III] or [6], the subject Descriptive Geometry seems to be restricted to standard constructions like the determination of the true length of a line segment or the intersection of two plane polygons in 3-space. From this point of view it must look rather strange that prominent geometers devoted their whole academic life to promote this subject.

2.1. Descriptive Geometry in Europe

In order to explain the meaning of 'Descriptive Geometry' in central Europe, let us look for definitions in German textbooks published in the last five decades:

- J. Krames defined in [9]: *"Descriptive Geometry is the high art of spatial reasoning and its graphic representation".*
  This definition has also been cited by R. Bereis in [1].
- H. Brauner took up a recommendation given by E. Kruppa and preferred the designation 'Constructional Geometry' [Konstruktive Geometrie] instead of Descriptive Geometry. He defined in [4]: *"Constructional Geometry encompasses the analysis of 3D objects by means of graphical or mathematical methods applied to 2D images."*
- F. Hohenberg, whose textbook [7] focusses on applications of Descriptive Geometry in technology, formulated: *"Constructional Geometry teaches how to grasp, to imagine, to design, and to draw geometrical shapes."*
- W.-D. Klix gives in his recent textbook [8] the following extended explanation: *"Descriptive Geometry is unique in the way how it promotes spatial reasoning, which is so fundamental for each creative activity of engineers, and how it trains the ability to express spatial ideas graphically so that they become understandable for anybody else."*

As a consequence, I would like to summarize in the following way.

Definition: *'Descriptive Geometry' is a method to study 3D geometry through 2D images. It provides insight into structure and metrical properties of spatial objects, processes and principles. Typical for Descriptive Geometry is the interplay*

a) between the 3D situation and its 2D representation, and

b) between intuitive grasping and rigorous logical reasoning.

According to this definition, Descriptive Geometry courses in central Europe cover not only projection theory, but also modeling techniques for curves, surfaces, and solids thus offering insight into a broad variety of geometric shapes. Besides, an intuitive approach to elementary differential-geometric properties of curves and surfaces and some 3D analytic geometry is included ([12, 13]). And in addition, one aim is also to develop and to refine the students' problem-solving skills.

2.2. G. Monge's definition

Gaspard Monge (1746–1818) is declared the founder of the science of Descriptive Geometry. This does not mean that he himself developed all the graphical methods. In contrary, most of them can already be found in earlier books, e.g., in those of Amédée François Frezier.

However, G. Monge was a most effective scientist and manager who spread his ideas of Descriptive Geometry with the publication of his *'Leçons de géométrie descriptive'* (1799) from France over whole Europe. We find in [10], p. 1, the following introductory statements:

"La Géométrie descriptive a deux objets:

- *le premier, de donner les méthodes pour représenter sur une feuille de dessin qui n'a que deux dimensions, savoir, longueur et largeur, tous les corps de la nature qui en ont trois, longueur, largeur et profondeur, pourvu néanmoins que ces corps puissent être définis rigoureusement.*
- *Le second objet est de donner la manière de reconnaître, d'aprés une description exacte, les formes des corps, et d'en déduire toutes les vérités qui résultent et de leur forme et de leurs positions respectives."*

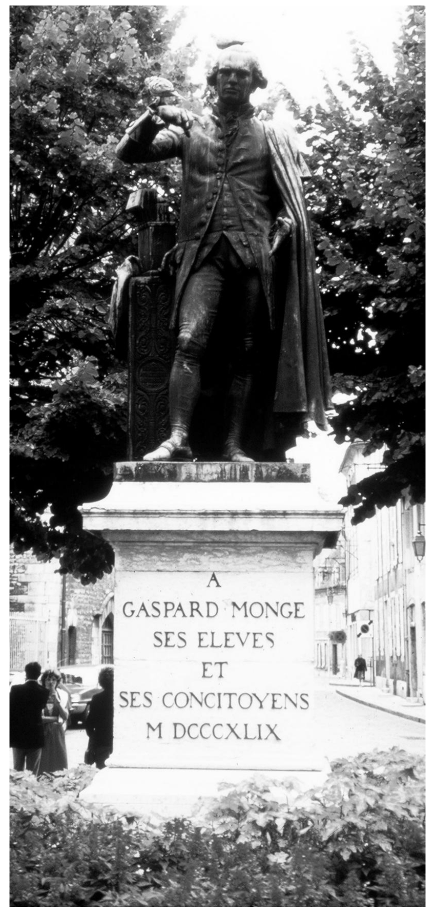

Fig. 1: Statue of G. Monge
Place de Monge, Beaune (birthplace)
Dep. Côte-d'Or, France

This proves that the two main objectives of Descriptive Geometry — imaging and analysing 3D objects — date back to its founder. These two targets can also be found in new encyklopedias like Brockhaus [5]:
"Descriptive Geometry = subject of mathematics, ... The aim of DG is the representation of 3D objects ... as well as the interpretation of given images ..."

2.3. The choice of the name

It is remarkable that the word *'drawing'* does not appear in Monge's definition. In Descriptive Geometry drawing[1] is the *guide to geometry* (compare [14]) but not the main aim;

[1] It is said that Felix Klein once stated: *"Among all mathematicians, geometers have the advantage to see what they are studying."*

This fixture is made from mild steel and consists of a rectangular block 75mm high, 44mm long and 100mm wide. It has a 25mm thick by 100mm wide flange protruding from the 100mm face of the block with the lower surfaces (base) aligned. The free end of the flange is rounded with a 50mm radius and at the centre of that radius is a hole 8mm diameter through the flange with a 20mm diameter counterbore 10mm deep in the top surface of the flange. The overall length of the fixture is 150mm.
The rectangular block has a Vee shaped slot symmetrically through the top surface in a longitudinal direction. It is 38mm each side of the centre at the top surface and is 45 degrees to this surface. The bottom of the Vee slot is removed by a rectangular slot 19mm wide with its bottom face 10mm above the top face of the flange.

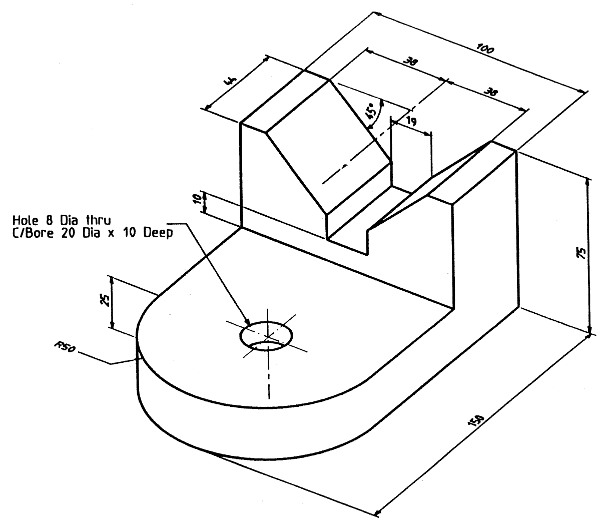

Fig. 2: On the importance of graphic representations — one illustration versus '1000 words'.
Source: K. Suzuki [15, Fig. 1] (with permission of Heldermann Verlag)

we teach geometry instead of construction techniques. Note that the French *'descriptive'* means 'describing', 'representing' but not necessarily 'graphically depicting'.

Nevertheless, in the public meaning Descriptive Geometry has falsely become synonymic for 'manually drawing images of 3D objects'. As in the last decades manual drawing with traditional instruments has been replaced by CAD or mathematical software with graphic output, 'people on the road' frequently conclude that therefore Descriptive Geometry has become obsolete.

However, this is totally wrong: In contrary,

- only people with a profound knowledge in Descriptive Geometry are able to make extended use of CAD programs as the communication is usually based on views only.
- The more powerful and sophisticated a modeling software, the higher the required geometric knowledge.
- A poor designer will never become perfect only by using CAD instead of traditional tools.

For similar reasons the importance of mathematics is still increasing though computers take over the computational labour.

Another misinterpretation of Descriptive Geometry is to consider it only as a theoretical, rather 'academic' subject. F. Hohenberg could disprove this opinion in his textbook [7] in a convincing way. In many examples he demonstrated application of Descriptive Geometry to real-world requirements.

In order to defend the true meaning of Descriptive Geometry, there were various attempts to rename this subject. Its applicability is stressed by using the names 'Technical Geometry' or 'Applied Geometry' instead of 'Descriptive Geometry'. As already mentioned, another choice is 'Constructive Geometry' — 'constructive' in its figurative sense. It should indicate that not only manual drawings but also mathematical computations are used in this subject.

Anyway, the original Monge definition of 'Descriptive Geometry' with its wide meaning covers all these aspects. So, in my opinion the original name is still appropriate. However, some find this name old-fashioned. For strategic reasons they are seeking for more attractive designations which make evident that temporary courses on Descriptive Geometry include also some methods from computer science like *'geometric modeling'* as well as *'visualization techniques'* and of course CAD-programs. In this sense *'Geometric Modeling and Visualization'* or more briefly *'Modeling and Imaging'* could be appropriate.

And for those who like to translate 'descriptive' by 'graphically depicting' only, I add the following statement: *'Descriptive Geometry' is more than 'descriptive' geometry as well as 'Geometry' is more than its literal sense, i.e., 'measuring the earth'.*

3. What should remain in the students' brain

In order to estimate the educational effect of any subject included in a curriculum, one should try to figure out what remains in the student's brain after all details are already forgotten. I would like to state that even for poor students the education in Descriptive Geometry brings about the ability

- to comprehend spatial objects from given principal views, and
- to specify and grasp particular views. Besides,
- the students get an idea of geometric idealization (abstraction), of the variety of geometric shapes, and of geometric reasoning.

The first two items look rather elementary. However, these intellectual abilities are so fundamental that many people forget later how hard these abilities are to achieve.

3.1. The importance of principal views

Familiarity with the principal views — *top view, front view, and side view* — are substantial for several reasons, e.g.,

- they are more or less abstract as they do not correspond to our personal visual impression. But abstraction simplifies.
- In the majority of cases they better make evident the essential properties of spatial structures, and
- inspecting these planar views is much easier than to concentrate on the original spatial structure.

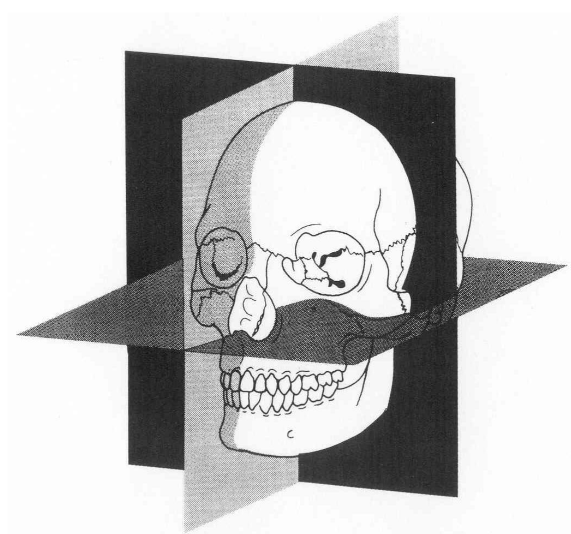

Fig. 3: Explanation of principal views in a textbook for dentists

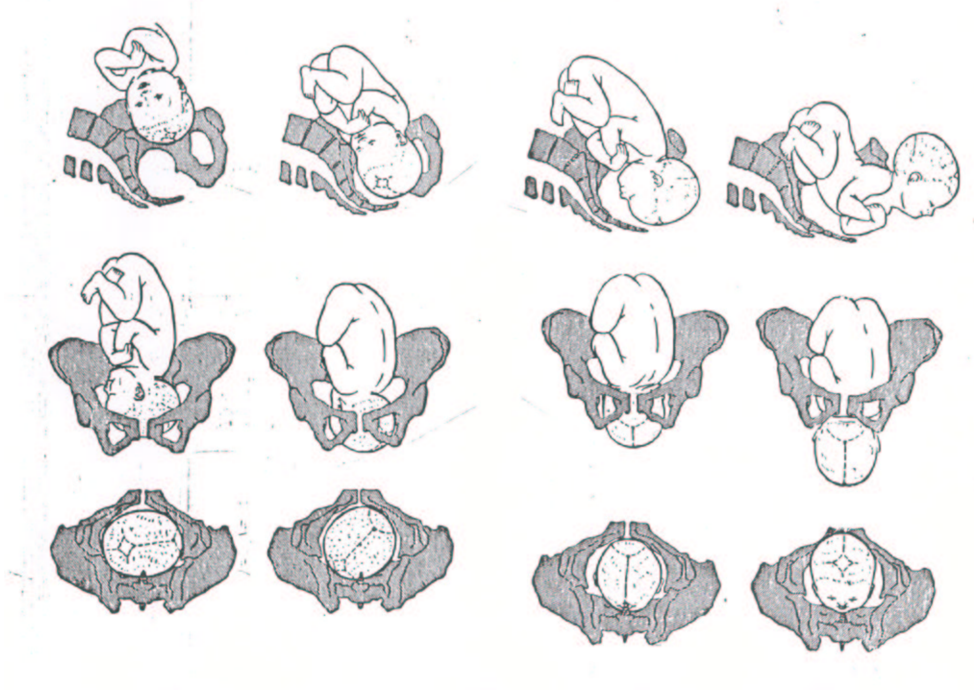

Fig. 4: Principal views for gynaecologists
There is no better way to explain the baby's 3D movement when being born[2]

However, it needs training to become familiar with this kind of representation and to grasp the shape of any 3D object just by looking at its principal views. Nobody questions the necessity of a permanent training for sportsmen. But in case of Descriptive Geometry, people often neglect this necessity and they speak of a purely academic subject, when, e.g., in introductory exercises two triangles in space are to intersect.

Medical doctors often hold in esteem their Descriptive Geometry education. In anatomy, they could much easier comprehend the course of blood-vessels or nerves just by sketching them in the principal views. And in orthopaedy, they were better able to grasp how human joints are operating and why mislocations have specific consequences.

A few months ago Austrian television was broadcasting a life operation at a human skull: The surgeon had to correct a mislocation of the cheek-bone, caused by a traffic accident. In a pre-operative step the required position of the cheek-bone was already marked on a screen. By an image-fusion this virtual posture was combined with the actual one. So, the surgeon's work consisted in making these two positions coincident by manual manipulations at the patient.

How did the surgeon control his work? He inspected the three principal views as they allowed to decompose the true 3D displacement into planar motions.

3.2. The art of specifying particular views

Axonometric views are important and well understandable for everybody. And they are appropriate to remember on a known object or to compare with a real object nearby. However, no angle, no length, no planar shape appears in true size. Orthogonality can be figured out only because of some additional assumptions based on experience or estimation. So, these views are never sufficient for a *'description exacte'* as required in Monge's

[2]The author is grateful to Prof. A. Schmid-Kirsch, University Hannover, for submitting this Fig. 4.

definition.[3]

For a detailed analysis of a 3D object *particular views* (auxiliary views) with planes in edge view or lines in point view really can reveal the spatial situation. Such views often are the key to the solution of a 3D problem. In my opinion these particular views make the sublime art of Descriptive Geometry. Only in Descriptive Geometry courses the students learn what conditions can be simultaneously fullfilled in particular views and how such views can be specified.

The following example (Fig. 5, cf. [11]) shall demonstrate the advantage of particular views:

Example: *Where does the sun rise earlier on June 21, in Oslo or in Vienna.*

city	*Eastern longitude*	*Northern latitude*
Oslo	$10,6°$	$59,9°$
Vienna	$16,4°$	$48,2°$

We specify a front view in Fig. 5 with sun rays parallel to the image plane. Then we assume that this view is taken in the moment when the sun is rising in Oslo on June 21. As soon as Vienna is displayed in this view, we see at first sight the solution of the posed question.

The same view is also useful for clearing additional and more detailed problems like the following:

a) Can it happen over the period of one year that the sun rises simultaneously in Oslo and Vienna?

b) We increase the precision by paying attention to the fact that because of *refraction* in the atmosphere the sun is still approx. $0,6°$ under the local horizon when for the observer on earth the sun seems to rise.

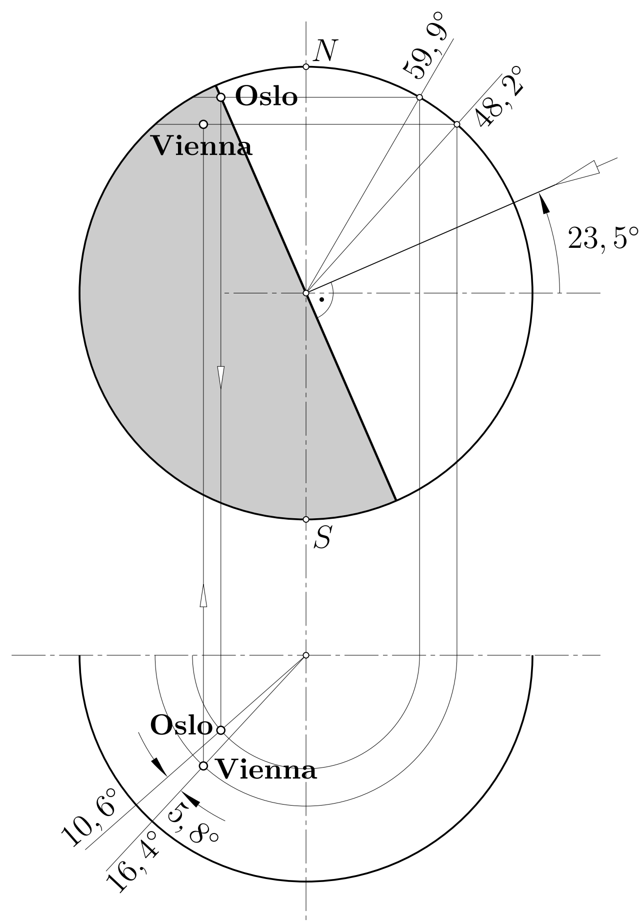

Fig. 5: Where does the sun rise earlier on June 21, in Oslo or in Vienna?

c) In the *zone of astronomic dawn* the sun is between $6°$ and $18°$ under the local horizon. Inspecting the particular view presented above, it is easy to comprehend why the period of the daily dawn is shorter when the observer is nearer to the equator.

[3]The same is true for pictures *shaded* like photographs. The can be very impressive, but also extremely cheating. Pure *line graphics* look less attractive; they are more abstract. But often this is an advantage as much more information is included, and line graphics allow to concentrate on that which is essential.

3.3. Views are a guide to spatial geometry

I don't know if anybody is able to manipulate virtual 3D objects — without any tools, in his imagination only — and to figure out how these objects look like in different postures. Maybe, sculptors or pilots have this mental ability. Actually, I myself don't; and the *rhombic dodecahedron* serves for me as a convincing example:

This convex polyhedron can be built by erecting quadratic pyramides with 45° inclined planes over each face of a cube (see Fig. 4). As any two coplanar triangles can be glued together forming a rhomb, this polyhedron has 12 congruent faces and seems to be well understood. Nevertheless, I'm not able to imagine (with closed eyes) how this polyhedron looks like from above when it is resting with one face on a table. Fortunately, a simple freehand sketch helps to figure out this view as well as other remarkable properties like the following:

- There are two types of vertices at the rhombic dodecahedron: 8 vertices belong to the initial cube; the other 6 are mirror images of the cube's center under reflection in the faces.
- The rhombic dodecahedron is the intersection of 3 quadratic prisms with pairwise orthogonal axes (see Fig. 7).

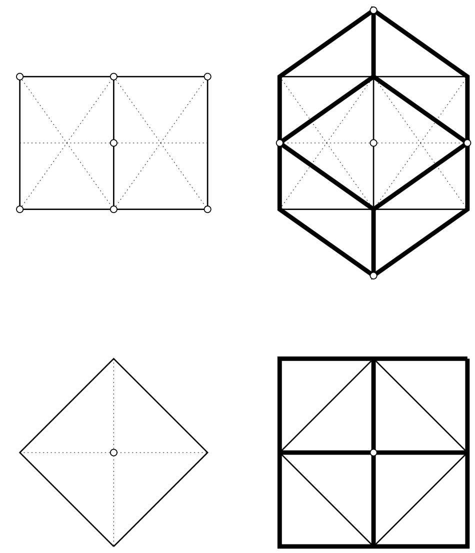

Fig. 6: Cube and rhombic dodecahedron

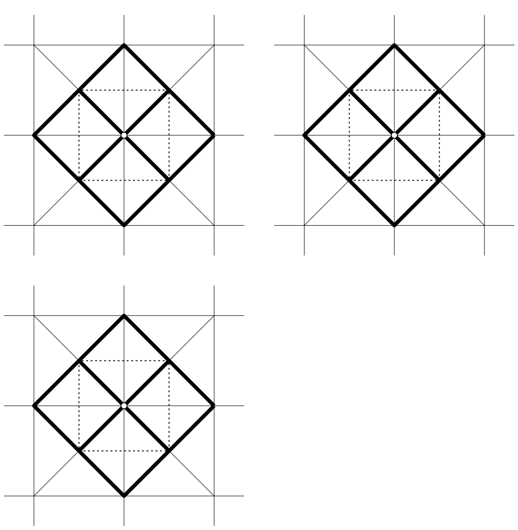

Fig. 7: Rhombic dodecahedron as the intersection of three quadratic prisms

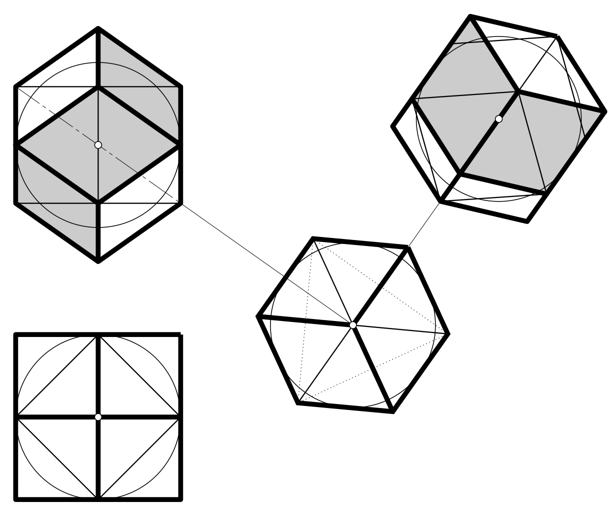

Fig. 8: Different views of the rhombic dodecahedron

- The rhombic dodecahedron is the intersection of hexagonal prisms with axes placed on cube-diagonals. There are chains of 6 adjacent faces (note shaded rhombs in Fig. 8) which are located on the same hexagonal prism.
- The side and back walls of a honey comb belong to a rhombic dodecahedron.

- Each dihedral angle makes 120°, and there is an in-sphere (contacting all edges of the initial cube).
- The rhombic dodecahedron[4] is dual to the cuboctahedron.
- The rhombic dodecahedron is a space-filling polyhedron. This can be figured out by starting with a '3D-chessboard' built from cubes. Then the 'white' cubes can be partitioned into 6 quadratic pyramides. Each can be added to the adjacent 'black' cube thus enlarging it to a rhombic dodecahedron.

4. Descriptive Geometry in presence of computers

● Descriptive Geometry
○ Computergraphics,
▲ Engineering Graphics,
△ 3D-CAD,
\+ control persons

Source: K. Suzuki [15, Fig. 5] (with permission of Heldermann Verlag)

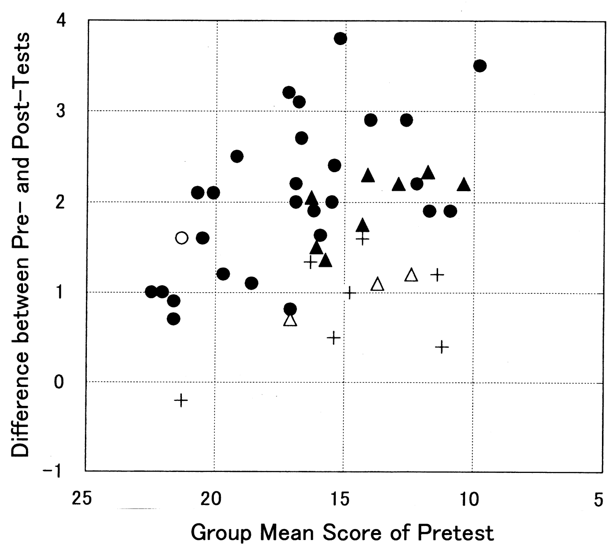

Fig. 9: Differences between pre- and post-MCT-test at Japanese students before and after the graphics education

The statistics above in Fig. 9 reveals the high effect of traditional Descriptive Geometry education in improving spatial abbility. However, one should permanently try to present the topics in actualized form so that they are attractive for the majority of young people. The following tables summarize some of these aspects:

What is obsolete:
• complicated manual constructions, • hard theoretical proofs, • the theory of how to obtain images of particular 3D objects

[4]More strictly, it should be called *first rhombic dodecahedron.* Due to S. Bilinski [3] there is a *second* one: In this case the dihedral angle is 144°. This polyhedron is obtained from the triacontahedron, the dual of the icosidodecahedron, by removing two prismatic zones and bringing the remaining pieces together. The author is grateful to H. Martini for pointing his attention to this fact.

What is still necessary:
• '3D-competence', i.e.,
• the capability to comprehend virtual 3D situations from given images,
• mental orientation in 3-space (e.g., user coordinate system),
• basic knowledge of 3D geometry,
• promoting creativity and problem-solving skills,
• applications of geometry,
• producing attractive illustrations.

But one must not forget that there are *additional demands* on Descriptive Geometry courses:

Additional demands:
• Handling software for geometric modeling and visualization,
• treating new geometric shapes (e.g., B-spline surfaces),
• competence in handling graphics files (in different format),
• design of animations.

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